(1) Let a, b be odd integers.

Prove that $\sqrt{a^2+b^2}$ is irrational.

Hint: Look at divisibility by the powers of 2.

(2) Prove that for any real numbers $a < b$ there exists an irrational number c such that $a < c < b$.

Hint: Look at the numbers of the form $q\sqrt{2}$ where q is rational.

(3) Show that the equation

$$3x^3 + 2x^2 - 5x - 2 = 0$$

has no rational solutions.

(4) Suppose $5 + 4i = (a + bi)(c + di)$ where a, b, c, d are integers. Prove that $|a + bi| = 1$ or $|c + di| = 1$.

Hint: use that $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$.

(5) Let $P(z) = a_n z^n + \ldots + a_1 z + a_0$ be a polynomial with real coefficients.

Prove that if z_0 is a root of $P(z) = 0$ then $\bar{z}_0$ is also a root of $P(z) = 0$.

(6) Prove that for any complex numbers z_1, z_2, z_3 we have

$$(z_1 z_2) z_3 = z_1 (z_2 z_3)$$