

A linear map $T: V \rightarrow V$ is called skew-adjoint if $T^* = -T$.

Prove that if T is skew-adjoint on a *real* vector space and $\dim V$ is odd, then T is singular (i.e. $\text{Null}(T) \neq 0$).