Consider the inner product on the space of $n \times n$ complex matrices given by $\langle A, B \rangle = trAB^*$.

a) Prove that $\|AA^*\| \leq \|A\|^2$ for any A.

b) Prove that $\|AB\| \leq \|A\| \cdot \|B\|$ for any matrices A, B.

Hint: use the fact that $tr(XY) = tr(YX)$ *for any* X, Y *(you'll have to prove this first).*

c) Suppose that a power series $f(z) = \Sigma_{n=0}^{\infty} a_n z^n$ converges absolutely for all z with $|z| < R$.

Without using Jordan forms prove that the power series $f(A) = \Sigma_{n=0}^{\infty} a_n A^n$ converges for any matrix A satisfying $\|A\| < R$.