(1) Let $f\colon [0,1] \to R$ be integrable on $[0,1]$. Let $F(x) = \int_0^x f(t)dt$. Show that F is continuous on $[0,1]$.

Hint: Use that f is bounded.

(2) Let $F(x) = \int_{2x}^{x^2} \sin(e^{tx})dt$.

Show that $F(x)$ is C^1 and find the formula for $F'(x)$.

(3) Give an example of a function $f\colon [0,1] \times [0,1] \to R$ such that f is not integrable but $\int_0^1 (\int_0^1 f(x,y)dy)dx$ exists.

(4) Let $Q = [0,1] \times [0,1]$ Let $f\colon Q \to R$ be defined as follows

$$f(x,y) = \begin{cases} 1/q \text{ if } x = p/q \text{ where p,q are positive integers with no common factor and } y \text{ is rational} \\ 0 \text{ otherwise} \end{cases}$$

(a) Show that $\int_Q f$ exists and compute it.

(b) Compute $\overline{\int}_{[0,1]} f(x,y)dy$ and $\underline{\int}_{[0,1]} f(x,y)dy$ for any $x \in [0,1]$.

(c) verify Fubini's theorem for f.

(5) Compute the volume of the ball of radius r in R^3.

Extra Credit. To be written up and handed in separately from the rest of the homework.

Let $f\colon R^n \to R^m$ be C^1 where $n < m$. Show that interior of $f(R^n)$ is empty.