(1) let $f\colon R^n \to R^n$ be a diffeomorphism. let $A \subset R^n$ be any subset.

Prove that $f(br(A)) = br(f(A))$.

(2) Let $n > 1$. Give an example of an $n \times n$ matrix A which preserves volume but is not orthogonal.

(3) Prove that isometries of R^n form a group under the composition.

(4) let V be a finite dimensional vector space over R. Let $e_1, \ldots, e_n$ be a basis of V. look at the dual elements $e_i^* \in V^*$ defined by

$$e_i^*(e_j) = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$$

Prove that $e_1^*, \ldots, e_n^*$ span V^*.

(5) prove that $\int_{(-\infty+\infty)}^{ext} e^{-x^2} dx = \sqrt{\pi}$.

Hint: Show that $(\int_{(-\infty+\infty)}^{ext} e^{-x^2} dx)(\int_{(-\infty+\infty)}^{ext} e^{-y^2} dy) = \pi$

(6) Let V be a finite dimensional vector space over R. Let $\langle \cdot, \cdot \rangle$ be an inner product on V. Let $\mathcal{L}(V, V)$ be the space of linear maps from V to V and $\mathcal{T}^2(V)$ be the space of 2-tensors on V. Consider the following map $\phi\colon \mathcal{L}(V, V) \to \mathcal{T}^2(V)$ given by the formula $\phi(A)(v, w) = \langle A(v), w \rangle$

(a) Prove that ϕ is an isomorphism.

(b) Prove that A is symmetric if and only if $\phi(A)$ is symmetric.

(c) Note that $T = \langle \cdot, \cdot \rangle$ is a 2-tensor. Find $\phi^{-1}(T)$.