(1) let T be a k-tensor on V. Prove that T is alternating if and only if $T(v_1, \dots v_k) = 0$ for any $v_1, \dots v_k$ such that $v_i = v_j$ for some $i \neq j$.

(2) Let T be a 2-tensor on V.

Prove that there exist unique tensors T_1, T_2 such that T_1 is alternating, T_2 is symmetric and $T = T_1 + T_2$

(3) let $\sigma \in S_{k+l}$ be the following permutation

$$\begin{pmatrix} 1 & \dots & k & k+1 & \dots & k+l \\ l+1 & \dots & l+k & 1 & \dots & l \end{pmatrix}$$

Prove that $sign(\sigma) = (-1)^{kl}$.

(4) Let $e_1, \dots e_n$ be a basis of a vector space V.

Let $T = (e_1^* + e_3^*) \otimes e_1^*$ and $S = (e_1^* + e_2^*) \otimes e_2^*$.

Compute $T \otimes S$ and $alt(T \otimes S)$.

Extra Credit. Compute the dimension of the space of symmetric k-tensors on a vector space V.

Suppose $\dim V > 1$ and $k > 2$. Is it true that any k tensor can be written as a sum of a symmetric and an alternating tensor?