(1) Let $f\colon R^3 \to R^3$ be given by $f(x,y,z) = (x+y\sin(2z), xe^z, \sqrt{1+x^2+y^2})$. Let $\omega = zdx \wedge dy$. Compute $f^*(\omega)$.

(2) Prove that if U is an open set in R^n star shaped with respect to a point $a \in U$ then any closed form on U is exact.

(3) Give a careful proof of the fact mentioned in class:

   if $f\colon R^n \to R^m$ and $g\colon R^m \to R^l$ are smooth then $(g \circ f)^* = f^* \circ g^*$.

(4) Give a careful proof of the following fact mentioned in the book.

   Let $\omega = -\frac{y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy$ be a form on $R^2 \backslash \{(0,0)\}$

   (a) Verify that $d\omega = 0$.

   (b) Give a careful proof of the following fact mentioned in the book: ω is not exact.

   Hint: Look at polar coordinates $x = r\cos\theta, y = r\sin\theta$ on $R^2 \backslash R_+$. Here $R_+ = \{(x,0) \text{ where } x \geq 0\}$, $r > 0, 0 < \theta < 2\pi$. Show that if $\omega = df$ then $f(r\cos\theta, r\sin\theta) = \theta + \mathit{const}$ in polar coordinates. obtain a contradiction with continuity of f on R_+.

Extra Credit: John Nash's Problem.

Is it true that every closed 1-form on $R^3 \backslash \{(0,0,0)\}$ is exact?