

- (1) Give an  $\epsilon$ - $\delta$  proof of the theorem mentioned in class:

Let  $f: R^n \rightarrow R^m$  and  $g: R^n \rightarrow R$  satisfy

$\lim_{x \rightarrow a} f(x) = 0$  and  $g(x)$  is bounded on  $B(a, \delta)$  for some  $\delta > 0$

then  $\lim_{x \rightarrow a} g(x)f(x) = 0$

- (2) Let  $f: R^n \rightarrow R^m$  satisfy  $\lim_{x \rightarrow 0} f(x) = L$  prove that for any line passing through the origin the limit of  $f(x)$  as  $x \rightarrow 0$  along this line exists and is equal to  $L$ . More precisely. For any vector  $x \in R^n$ ,  $x \neq 0$  consider the line  $t \cdot x$  where  $t \in R$ . Prove that

$$\lim_{t \rightarrow 0} f(tx) = L$$

**Extra Credit Problem (to be written up and submitted separately)**

Give an example of a function  $f: R^2 \rightarrow R$  such that  $\frac{\partial f}{\partial x}(x, y)$  and  $\frac{\partial f}{\partial y}(x, y)$  exist everywhere but  $f(x, y)$  is not differentiable at  $(0, 0)$ .