

- (1) Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  be given by

$$f(x, y) = \begin{cases} \sqrt{|xy|} & \text{if } x \geq 0 \\ -\sqrt{|xy|} & \text{if } x < 0 \end{cases}$$

Show that  $D_h f((0, 0))$  exists for any  $h \in \mathbb{R}^2$  but  $f$  is not differentiable at  $(0, 0)$ .

- (2) Let  $x(t_1, t_2) = t_1 e^{t_2}$ ,  $y(t_1, t_2) = t_1^2 + \sin(t_1 t_2)$ . Let  $f(x, y)$  be a differentiable function  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ . Let  $g(t_1, t_2) = f(x(t_1, t_2), y(t_1, t_2))$ . Express  $\frac{\partial g}{\partial t_1}(1, 0)$  and  $\frac{\partial g}{\partial t_2}(1, 0)$  in terms of partial derivatives of  $f$ .