(1) Let $f(r,\theta) = (r\cos\theta, r\sin\theta)$ be a map $f\colon U \to R^2$ where $U = \{(r.\theta)|r > 0, -\pi/2 < \theta < \pi/2\}$.

Let $g(x,y) = (\sqrt{x^2+y^2}, \arctan(y/x))$ be a map $g\colon V \to R^2$ where $V = \{(x,y)|x > 0\}$.

Show that $f\colon U \to V$ is a bijection with $f^{-1} = g$. Compute $df(r,\theta), dg(x,y)$ and verify that $[dg(f(r,\theta)] = [df(r,\theta)]^{-1}$.

(2) Let $f = f^1(x,y), f^2(x,y))\colon R^2 \to R^2$ be a C^1 map satisfying $f(x,0) = (\cos x, x^2)$. Suppose f^{-1} exists. can it be differentiable at $(1,0)$?