

- (1) Let $f: R^n \rightarrow R^m$ be C^2 where $n > m$. as usual we denote coordinates in R^n by (x, y) where $x \in R^{n-m}, y \in R^m$. Let $p = (x_0, y_0) \in R^n$ and $c = f(p)$. Suppose the matrix $\frac{\partial f}{\partial y}(p)$ is invertible. Let $y(x)$ be the implicit function given by solving $f(x, y(x)) = c$ near p . Show that $y(x)$ is C^2 .
- (2) let $f: R^3 \rightarrow R^2$ be given by $f_1(x, y, z) = x + 2y + \sin(z + y)$, $f_2(x, y, z) = x + e^{xy} + z$. Show that the level set $\{f_1 = 0, f_2 = 1\}$ can be solved near $(0, 0, 0)$ as $x = x(z), y = y(z)$ and compute $\frac{\partial x}{\partial z}(0)$ and $\frac{\partial y}{\partial z}(0)$.