(1) Let $f\colon Q \to \mathbb{R}$ be integrable.
Prove that $|f$ is integrable and $|\int_Q f| \leq \int_Q |f|$.

(2) Let $p \in \mathbb{R}^n$ and $B(p,R) \subset \mathbb{R}^n$ be the ball of radius R centered a p. Let $f\colon B(p,R) \to \mathbb{R}$ be C^1 with $|D_i f(x)| \leq C$ for any $i = 1, \dots n$ and any $x \in B(p,R)$.
Prove that f is uniformly continuous on $B(p,R)$.

(3) seam
   (a) Let $f(x) = \int_{x^2}^{3x} \sqrt{t^3 + x^3} dt$ Find the expression for $f'(x)$.
   You **DO NOT** need to evaluate the integral in that expression.
   (b) Let $f(x) = \int_0^{h(x)} (g(x,t))^4 dt$
   where h and g are C^1.
   Find the formula for $f'(x)$.
   (c) Let $f(x) = \int_a^{\int_b^x g(x,y)dy} g(x,y)dy$ where g is C^1.
   Find the formula for $f'(x)$.

(4) Let $f\colon [0,1] \to [01]$ be integrable. Consider the following function $f\colon Q = [0,1] \times [0,1] \to \mathbb{R}$

$$F(x,y) = \begin{cases} 1 \text{ if } y < f(x) \\ 0 \text{ if } y \geq f(x) \end{cases}$$

Prove that F is integrable over Q and $\int_Q F = \int_0^1 f$.

(5) Let $f\colon [0,1] \times [0,4] \to \mathbb{R}$ be given by

$$f(x,y) = \begin{cases} xy^2 \text{ if } y < x^2 \\ x + 2y \text{ if } y \geq x^2 \end{cases}$$

Verify that f is integrable and compute $\int_Q f$ in two different ways using Fubini's theorem.

(6) Let Q be a rectangle in $\mathbb{R}^n$. Let $S \subset Q$. consider the characteristic function of S on Q given by

$$\chi_S(x) = \begin{cases} 1 \text{ if } x \in S \\ 0 \text{ if } x \notin S \end{cases}$$

prove that $\chi_S(x)$ is integrable if and only if $bd(S)$ has measure 0.
Hint: Show that $bd(S)$ is the set of points of discontinuity of $\chi_S(x)$.

Extra Credit Problem (to be written up and submitted separately)

Problem 3c on page 103.