MAT 257Y **Practice Term Test 1**

(1) Find the partial derivatives of the following functions
(a) $f(x, y, z) = \sin(x \sin(y \sin z))$
(b) $f(x, y, z) = x^{yz^2}$

(2) give an example of a nonempty set A such that the set of limit points of A is the same as the set of boundary points of A.

(3) Let $A, B \subset R^n$ be compact.
Prove that the set $A + B = \{a + b | a \in A, b \in B\}$ is compact.

(4) show that the intersection of arbitrary collection of closed sets is closed.

(5) show that $f\colon R^n \to R^m$ is continuous if and only if $f^{-1}(A)$ is closed for any closed $A \subset R^m$.

(6) Let R^{n^2} be the space of all $n \times n$ matrices. Consider the map $f\colon R^{n^2} \to R^{n^2}$ given by the formula
$f(A) = A \cdot A^T$.
Here A^T means the transpose of A.
Show that f is differentiable everywhere and compute $df(A)$.
Hint: use that $df(A)(X) = D_X f(A)$.

(7) Let $f = (f_1, f_2)\colon R^2 \to R^2$ be given by the formula $f_1(x, y) = x + y + y^3 + 1$, $f_2(x, y) = xe^y + 2$
Show that there an open set U containing $(0, 0)$ such that $f\colon U \to f(U)$ is a bijection and f^{-1} is differentiable on $f(U)$ and compute $df^{-1}(1, 2)$.

(8) Let $f(x, y) = x^y$ be defined on $U = \{(x, y) | x > 0\}$.
Verify that
$$\frac{\partial^2 f}{\partial x \partial y}(x, y) = \frac{\partial^2 f}{\partial y \partial x}(x, y)$$