MAT 257Y **Practice Term Test 2**

(1) Let $f\colon R^n \to R^m$ be C^1 where $n > m$. Suppose $[df(x_0)]$ has rank m.

Show that there exists $\epsilon > 0$ such that for any $y \in B(f(x_0), \epsilon)$ there exists $x \in R^n$ such that $f(x) = y$.

(2) Let A be a rectangle in R^n and let $S \subset A$ be a set of measure zero which is Jordan measurable. Show that S has content zero.

Hint: Use that $\int_A \chi_S$ exists and must be equal to zero.

(3) Let $f\colon [0,1] \times [0,1] \to R$ be continuous.

Show that

$$\int_0^1 (\int_0^x f(x,y)dy)dx = \int_0^1 (\int_y^1 f(x,y)dx)dy$$

(4) Let $f\colon R^2 \to R$ be C^2.

Prove that $F(x) = \int_0^1 f(x,y)dy$ is C^2 on R.

(5) Prove that the union of countably many sets of measure zero has measure 0.

(6) let $S = \{(x,y) \in R^2 | \quad x^2 + y^2 \leq 1, y \geq |x|\}$.

Compute $\int_S y$.

(7) Let $A \subset R^n, B \subset R^m$ be rectangles. let $f\colon A \times B \to R$ be integrable.

Prove that there is a set $S \subset A$ of measure 0 such that for any $x \in A \backslash S$ the integral $\int_B f(x,y)dy$ exists.

(8) Let $f\colon [-1,1] \times [-1,1] \to R$ be a continuous function. Suppose $f(-x,y) = -f(x,y)$ for any x, y.

Prove that $\int_{[-1,1]\times[-1,1]} f = 0$.

Hint: use Fubini's theorem.