MAT 257Y Practice Term Test 3

(1) Let A be a Jordan measurable set.

Prove that for any $\epsilon > 0$ there exists a compact Jordan measurable set $C \subset U$ such that $\int_{A \setminus C} 1 < \epsilon$.

Hint: Consider the lower Riemann sum for $\int_A 1$.

(2) Let ϕ_i be a partition of unity on an open set U. let $K \subset U$ be a compact set.

Prove that all but finitely many ϕ_i vanish on K.

(3) Let $c\colon [0,1] \to (R^n)^n$ be continuous. Suppose that $c^1(t), \ldots, c^n(t)$ is a basis of R^n for any t.

Prove that $(c^1(0), \ldots, c^n(0))$ and $(c^1(1), \ldots, c^n(1))$ have the same orientation.

(4) Let C be the triangle in R^2 with vertices $(0,0), (1,2), (-1,3)$

Compute $\int_C x + y$.

Hint: use a linear change of variables.

(5) Let $T \in \mathcal{T}^2(V)$.

Prove that $Alt(T) = 0$ if and only if T is symmetric. Is the same true if $T \in \mathcal{T}^3(V)$?

(6) Let $T \subset \mathcal{T}^2(V)$. Let T_{ij} be coordinates of T with respect to basis $e_1, \ldots, e_n$ and $\tilde{T}_{ij}$ be coordinates of T with respect to basis $\tilde{e}_1, \ldots, \tilde{e}_n$. Let A be the transition matrix from e to $\tilde{e}$.

Prove that $[\tilde{T}] = A^t[T]A$.

(7) Let $f\colon R^n \to R^n$ be a C^∞ diffeomorphism. Let $\omega = dx^1 \wedge \ldots \wedge dx^n$. Suppose $f^*\omega = \omega$.

Prove that $\mathrm{vol} U = \mathrm{vol} f(U)$ for any bounded open set U.

(8) Let $f\colon R^2 \to R^3$ be given by $f(x,y) = (x^2 + \cos(xy), e^{2xy}, xy^2)$. let $\omega = e^{xy} dx \wedge dz + 2x dy \wedge dz - \sin(xy) dy \wedge dz$

Compute $f^*\omega$.