Proof By Counterexample

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This proof structure allows us to prove that a property is not true by providing an example where it does not hold.

For example, to prove that "not all triangles are obtuse", we give the following counter example: the equilateral triangle having all angles equal to sixty. In this case, there are infinitely many counterexample. However, it only takes one.

Another example:

Claim. It is not the case that all linear functions in one variable are perpendicular to one another.

Proof. To prove that this is true, we need to find a pair of linear functions in one variable that are not perpendicular. We suggest the following counterexample:

$$f_1(x) = 3x + 4$$
$$f_2(x) = 2x - 1$$

In order to see that these two linear functions are not perpendicular, we notice that the slop of the first function is 3. Therefore, the slope of a perpendicular line must be $\frac{-1}{3}$. However, the slope of f_2 is 4, not $\frac{-1}{3}$. Therefore, they are not perpendicular, thereby proving that not all linear functions in one variable are perpendicular to one another. $\square$

<u>Note:</u> When proving by counterexample, it is not enough to state the counterexample. One must also explain why it is a counterexample.