MAT 137Y: Calculus with proofs
Assignment 9
Due on Thursday, March 25 by 11:59pm via Crowdmark

Instructions:

- You will need to submit your solutions electronically via Crowdmark. See MAT137 Crowdmark help page for instructions. Make sure you understand how to submit and that you try the system ahead of time. If you leave it for the last minute and you run into technical problems, you will be late. There are no extensions for any reason.
- You may submit individually or as a team of two students. See the link above for more details.
- You will need to submit your answer to each question separately.
- This problem set is about Unit 13.

1. (a) Prove the following Theorem

   Theorem 1. Let $\sum_{n=1}^{\infty} a_n$ be a series.

   $$\text{IF } \begin{cases} \lim_{k\to\infty} \sum_{n=1}^{2k} a_n & \text{exists} \\ \lim_{n\to\infty} a_n = 0 \end{cases}$$

   THEN the series $\sum_{n=1}^{\infty} a_n$ is convergent.

   Hint: You may use results from past assignments. They will make things simpler.

   (b) Prove that the theorem is false if we remove any one of the two hypotheses.

2. Let $\sum_{n}^{\infty} a_n$ be a CONVERGENT, NON-NEGATIVE series. Let f is a continuous function with domain $\mathbb{R}$. Decide whether each of the following series must be convergent, must be divergent, or we do not have enough information to decide. Prove it.

   (a) $\sum_{n}^{\infty} (n^n \cdot a_n)$

   (b) $\sum_{n}^{\infty} \ln(2 + a_n)$

   (c) $\sum_{n}^{\infty} \ln \frac{2 + a_{n+1}}{2 + a_n}$

   (d) $\sum_{n}^{\infty} \ln(1 + a_n)$

   (e) $\sum_{n}^{\infty} (-1)^n \sqrt{a_n}$

   (f) $\sum_{n}^{\infty} (a_n\, f(\sin n))$