MAT137 - Calculus with proofs

- **Deadline to add/change courses:**
  Wednesday, September 23

- TODAY: Definitions and proofs

- NEXT CLASS: Proof by induction
  - **Required videos: 1.14, 1.15**

One-to-one functions

Let f be a function with domain D.

f is *one-to-one* means that ...

- ... different inputs (x) ...
- ... must produce different outputs ($f(x)$).

Write a formal definition of "one-to-one".

One-to-one functions

Definition: Let f be a function with domain D.
f is one-to-one means ...

1. $f(x_1) \neq f(x_2)$
2. $\exists x_1, x_2 \in D,\ f(x_1) \neq f(x_2)$
3. $\forall x_1, x_2 \in D,\ f(x_1) \neq f(x_2)$
4. $\forall x_1, x_2 \in D,\ x_1 \neq x_2,\ f(x_1) \neq f(x_2)$
5. $\forall x_1, x_2 \in D,\ x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$
6. $\forall x_1, x_2 \in D,\ f(x_1) \neq f(x_2) \implies x_1 \neq x_2$
7. $\forall x_1, x_2 \in D,\ f(x_1) = f(x_2) \implies x_1 = x_2$

One-to-one functions

Let f be a function with domain D.

What does each of the following mean?

1. $f(x_1) \neq f(x_2)$
2. $\exists x_1, x_2 \in D,\ f(x_1) \neq f(x_2)$
3. $\forall x_1, x_2 \in D,\ f(x_1) \neq f(x_2)$
4. $\forall x_1, x_2 \in D,\ x_1 \neq x_2,\ f(x_1) \neq f(x_2)$
5. $\forall x_1, x_2 \in D,\ x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$
6. $\forall x_1, x_2 \in D,\ f(x_1) \neq f(x_2) \implies x_1 \neq x_2$
7. $\forall x_1, x_2 \in D,\ f(x_1) = f(x_2) \implies x_1 = x_2$

Proving a function is one-to-one

Definition

Let f be a function with domain D.
We say f is one-to-one when

- $$\forall x_1, x_2 \in D,\ x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$$
- OR, equivalently, $\forall x_1, x_2 \in D,\ f(x_1) = f(x_2) \implies x_1 = x_2$

Suppose I give you a specific function f and I ask you to prove it is one-to-one.

- Write the structure of your proof (how do you begin? what do you assume? what do you conclude?) if you use the first definition.
- Write the structure of your proof if you use the second definition.

Exercise

Prove that $f(x) = 3x + 2$, with domain $\mathbb{R}$, is one-to-one.

Proving a function is NOT one-to-one

Definition

Let f be a function with domain D.
We say f is one-to-one when

- $$\forall x_1, x_2 \in D,\ x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$$
- OR, equivalently, $\forall x_1, x_2 \in D,\ f(x_1) = f(x_2) \implies x_1 = x_2$

Suppose I give you a specific function f and I ask you to prove it is not one-to-one. You need to prove f satisfies the *negation* of the definition.

- Write the negation of the first definition.
- Write the negation of the second definition.
- Write the structure of your proof.

Exercise

Prove that $f(x) = x^2$, with domain $\mathbb{R}$, is not one-to-one.