# MAT137 - Calculus with proofs

- Assignment #2 due on October 15.

- TODAY: Limit laws and more proofs with limits

- WEDNESDAY: Squeeze theorem and more proofs with limits (**Watch videos 2.12, 2.13**)

# Indeterminate form

Let $a \in \mathbb{R}$. Let $f$ and $g$ be positive functions defined near $a$, except maybe at $a$.
Assume $\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$.

What can we conclude about $\lim_{x \to a} \frac{f(x)}{g(x)}$ ?

1. The limit is 1.
2. The limit is 0.
3. The limit is $\infty$.
4. The limit does not exist.
5. We do not have enough information to decide.

# True or False?

Is this claim true?

## Claim

Let $a \in \mathbb{R}$.

Let $f$ and $g$ be functions defined near $a$.

- IF $\lim_{x \to a} f(x) = 0$,
- THEN $\lim_{x \to a} [f(x)g(x)] = 0$.

# A new theorem about products

## Theorem

Let $a \in \mathbb{R}$. Let $f$ and $g$ be functions with domain $\mathbb{R}$, except possibly $a$. Assume

- $\lim_{x \to a} f(x) = 0$, and
- $g$ is bounded. This means that

$$\exists M > 0 \text{ s.t. } \forall x \neq a, |g(x)| \leq M.$$

THEN $\lim_{x \to a} [f(x)g(x)] = 0$

1. Write down the formal definition of what you want to prove.
2. Write down what the structure of the formal proof.
3. Rough work.
4. Write down a complete formal proof.

# Critique this "proof" – #1

- WTS $\lim_{x \to a} [f(x)g(x)] = 0$. By definition, WTS:
$$\forall \varepsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \implies |f(x)g(x)| < \varepsilon$$
- Let $\varepsilon > 0$.
- Use the value $\frac{\varepsilon}{M}$ as "epsilon" in the definition of $\lim_{x \to a} f(x) = 0$
$$\exists \delta_1 \in \mathbb{R} \text{ s.t. } 0 < |x - a| < \delta_1 \implies |f(x)| < \frac{\varepsilon}{M}.$$
- Take $\delta = \delta_1$.
- Let $x \in \mathbb{R}$. Assume $0 < |x - a| < \delta$
- Since $\exists M > 0$ s.t. $\forall x \neq 0, |g(x)| \leq M$
$$|f(x)g(x)| < \frac{\varepsilon}{M} \cdot M = \varepsilon.$$