MAT137 - Calculus with proofs

- Practice Test: Friday (today) 3pm to Saturday 3pm

- TODAY: Computations

- MONDAY: EVT and IVT **(Videos 2.21, 2.22)**
- WEDNESDAY: Derivatives (Videos 3.1, 3.2, 3.3)

Is this rigorous?

In Video 2.19 I explained that, since we know

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

we can also conclude

$$\lim_{x \to 0} \frac{\sin(2x)}{2x} = 1$$

and I called this a “change of variable” $(u = 2x)$

Why is this true?
Didn't we say there isn't a “composition law” for limits?

Transforming limits

The only thing we know about the function g is that $\lim_{x\to 0} \frac{g(x)}{x^2} = 2$. Use it to compute the following limits:

1. $\lim_{x\to 0} \frac{g(x)}{x}$
2. $\lim_{x\to 0} \frac{g(x)}{x^4}$
3. $\lim_{x\to 0} \frac{g(3x)}{x^2}$

Computations!

1. $\lim_{x\to 1} \left(x^2 + 2^x\right)$
2. $\lim_{h\to 2} \dfrac{h^3 - 5h^2 + 3h + 6}{h^3 - h^2 - 3h + 2}$
3. $\lim_{x\to 0} \dfrac{x}{3 - \sqrt{x+9}}$
4. $\lim_{x\to\infty} \dfrac{x^3 + 2x + 1}{4x^3 - x^2 + 6}$
5. $\lim_{x\to\infty} \dfrac{\sqrt{x^2+1} + 2x}{5x}$
6. $\lim_{x\to 0} \dfrac{x}{\sin(4x)}$
7. $\lim_{x\to 0} \dfrac{\tan^2(2x^2)}{x^4}$
8. $\lim_{z\to 0} \dfrac{\sin(2z^2)}{\cos(3z)\sin^2(5z)}$
9. $\lim_{x\to 0} \dfrac{\sin e^x}{e^x}$
10. $\lim_{x\to 0} \dfrac{1 - \cos x}{x}$
11. $\lim_{y\to 1} \dfrac{\sqrt{y+4} - \sqrt{4y+1}}{\sqrt{y} - 1}$
12. $\lim_{x\to\infty} \left[x - \sqrt{x^2 + x}\right]$