# MAT137 - Calculus with proofs

- Assignment #3 due on November 5
- Assignment #4 due on November 26

- TODAY: Functions and inverse functions

- FRIDAY: Exponentials and logarithms
  - **Watch videos 4.5, 4.7, 4.8, 4.9**
  - Supplementary videos: 4.6, 4.10, 4.11

# Fill in the Blanks

Assume that $f$ is an invertible function. Fill in the blanks.

1. If $f(-1) = 0$, then $f^{-1}(\_\_\_\_\_) = \_\_\_\_\_$.
2. If $f^{-1}(2) = 1$, then $f(\_\_\_\_\_) = \_\_\_\_\_$.
3. If $(2, 3)$ is on the graph of $f$, then _____ is on the graph of $f^{-1}$.
4. If $(2, 3)$ is on the graph of $f^{-1}$, then _____ is on the graph of $f$.

# Where is the error?

- We know that $\boxed{(f^{-1})' = \frac{1}{f'}}$
- Let $f(x) = x^2$, restricted to the domain $x \in (0, \infty)$

$$f'(x) = 2x \quad \text{and} \quad \boxed{f'(4) = 8}$$

- Then $f^{-1}(x) = \sqrt{x}$

$$(f^{-1})'(x) = \frac{1}{2\sqrt{x}} \quad \text{and} \quad \boxed{(f^{-1})'(4) = \frac{1}{4}}$$

- But $\boxed{(f^{-1})'(4) \neq \frac{1}{f'(4)}}$

# Derivatives of the inverse function

Let $f$ be a one-to-one function.
Let $a, b \in \mathbb{R}$ such that $b = f(a)$.

1. Obtain a formula for $\left(f^{-1}\right)'(b)$ in terms of $f'(a)$.

   *Hint:* This appeared in Video 4.4

   Take $\dfrac{d}{dy}$ of both sides of $\quad f(f^{-1}(y)) = y$.

2. Obtain a formula for $\left(f^{-1}\right)''(b)$ in terms of $f'(a)$ and $f''(a)$.

3. *Challenge:* Obtain a formula for $\left(f^{-1}\right)'''(b)$ in terms of $f'(a)$, $f''(a)$, and $f'''(a)$.

# Composition of one-to-one functions

Assume for simplicity that all functions in this problem have domain $\mathbb{R}$. Prove the following theorem.

**Theorem A**

Let $f$ and $g$ be functions.
IF $f$ and $g$ are one-to-one,
THEN $f \circ g$ is one-to-one.

Suggestion:

1. Write the definition of what you want to prove.
2. Figure out the formal structure of the proof.
3. Complete the proof (use the hypotheses!)