MAT 137Y: Calculus with proofs
Test 2 - Comments and common errors

Q1

- You **may not** use L'Hôpital's Rule. The quotient in this limit does not satisfy the hypotheses of L'Hôpital's Rule. Even if you did not check they hypotheses (why didn't you?) you should have noticed that we cannot write $f'(x)$ becasue f is only differentiable at 0.

  If you use L'Hôpital's Rule, then your solution is entirely wrong and you get 0 points.

  Besides, the test only covered Units 3 to 5. You should not expect to find a shortcut to a question using L'Hôpital's Rule, which we only learned in Unit 6.

- When you use the limit law for product, each individual limit must exist. You know one of the limits exists because you know f is differentiable at 0. Say so explicitly.

- We know $f'(0)$ exists, but we do not know that $f'(0) = 0$. It isn't.

Q2

- You are trying to prove that

$$\exists M \in \mathbb{R} \text{ such that } \forall x, y \in [a, b],\ |f(x) - f(y)| \leq M|x - y|$$

  Your choice of M **may not** depend on x or y. It can only depend on f, a, and b. Think back to what you learned about statements with 2 quantifiers in Unit 1 (Video 1.4)

  If you chose a value of M depending on x or y, then your proof is entirely wrong and you get 0 points.

  Think about proof structure. The most reasonable way to write your proof is

  - FIRST say what M is. That guarantees you are not choosing M depending on x or y.
  - SECOND fix arbitrary values of x and y.
  - Then go ahead and prove that $|f(x) - f(y)| \leq M|x - y|$.

- Do not assume that f' is positive. Picking as M the maximum of f' on $[a, b]$ may not work, because f' may take both positive and negative values. See the discussion in the sample solutions.

- When you invoke EVT or MVT, specify the function and the interval.

- When you use EVT or MVT, verify the hypotheses first.