MAT 137Y: Calculus with proofs
Test 3 - Part B
Comments and common errors

General comment

Don't bluff. Some students did not know what to do with one of the questions. That is okay. However, instead of skipping the question or answering only what they knew, some students proceeded to write lots and lots of nonsense in the hopes that some of it would give them partial marks by accident. Please do not do that. Have some dignity. It is transparently obvious when you do so, you don't get any points, and you make the TAs want to take on drinking.

Notice that bluffing is very different from making errors or leaving an incomplete answer. Making an error is okay. Not knowing an answer is okay. Being dishonest is not.

Q1

- We asked you to prove that a limit-indeterminate form of type $0 \cdot \infty$ can be *any* real number, not just that it can be some number.
- Some of you tried to prove something for generic functions f and g, instead of building specific examples. That does not make sense in this context. Please review Video 6.11, time 1:20 to 3:00.

Q2

- To use the LUB principle you need to prove that a set is both bounded above *and* non-empty.
- To prove the set of lower sums is bounded above you need to find one single number x such that

  $$\forall \text{ partitions } P, \; L_P(f) \leq x$$

  The number x **cannot** depend on P. It must be the same number for all partitions. If you wrote something like

  $$\forall \text{ partitions } P, \; L_P(f) \leq U_P(f)$$

  then you have not found an upper bound.
- Some of you assumed $\underline{I}_a^b(f)$ exists and proceeded to "prove" that it is equal to the supremum of the set of lower sums. That is not what we asked you, and it also makes no sense! If you assume $\underline{I}_a^b(f)$ exists, then by definition it *is* the supremum of the set of lower sums: there is nothing to prove!
- You may not assume f to be integrable (and also, it is irrelevant to the question).

Q3

- In the final step of your calculation, you need to use that f is continuous at 0 to be able to write $\lim_{x\to 0} f(x) = f(0)$.

  Say so explicitly. No, don't try to tell us it was obvious. You did not even notice that you were using continuity. You would have done the same manipulation even if f were not continuous (and then it would have been wrong).

- Explain what you are doing! In particular, when you use FTC or L'Hôpital's Rule, say so. Before using L'Hôpital's Rule, verify that you have an indeterminate form.

- Review the statement of FTC-1. When you use that

$$\frac{d}{dx}\int_a^x f(t)dt \;=\; f(x)$$

  there cannot be an "x" inside the integral. Instead, use the product rule if necessary.

- $\left(\int fg\right) \neq \left(\int f\right)\left(\int g\right)$

- f is not differentiable. If your solution included f', it can't possibly be right.