MAT 137Y: Calculus with proofs
Test 5 - Part B
Comments and common errors

Q1

- 0^0 is undefined. It is neither 0, nor 1. However, when we write x^0 in a power series, we are using a convention. The infinite sum

$$\sum_{n=0}^{\infty} c_n x^n$$

represents the infinite sum

$$c_0 + x_1 x + x_2 x^2 + c_3 x^3 + \dots$$

This is the convention we have used pretty much *every time* we write a power series, so you have definitely encountered it before. For example, when we write

$$e^x = \sum_{n=0} \frac{x^n}{n!}$$

we actually mean

$$e^x = \sum_{n=0} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

- The equation of the tangent line is

$$y = \frac{1}{2}x + 1.$$

It is *not* any of the following

$$\frac{1}{2}x + 1$$
$$L(x) = \frac{1}{2}x + 1$$
$$P_1(x) = \frac{1}{2}x + 1$$

Q2

- n is a dummy index. You cannot "take it out of the sum". You can also not use n for something both outside the sum and as the summation index. And you cannot cancel the n in the sum with an n "outside of the sum".
- If you just estimate the series, then you have not computed it.
- If you leave your final answer as a series, you probably have not done anything.

Q3

- n is a dummy index. You cannot "take it out of the sum". You can also not use n for something both outside the sum and as the summation index. And you cannot cancel the n in the sum with an n "outside of the sum".
- You need to explain why n is the largest value for which the limit exists. In other words, why the limit does not exists for values of $n > 4$.
- You cannot conclude the limit is $\pm\infty$ for values of $n > 4$, but you can conclude that each side limit (on the left and on the right) is $\pm\infty$. And then you can conclude the limit does not exist.
- If you use L'Hôpital's Rule, you need to explain you are doing. You will need a particular justification for why the limit does not exists for $n > 4$, and overall the explanation is likely to be too convoluted.
- You may not replace $\cos x$ with 1 when computing the limit.