# APM426: GENERAL RELATIVITY

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**(1) special relativity**
- **(a) spacetime structure**
  - **(i) light speed invariance; spacetime interval**
  - **(ii) light cone**
  - **(iii) Lorentz transformations; Poincaré group**
  - **(iv) proper time/length**
  - **(v) paradoxes and their resolution**
- **(b) mechanics**
  - **(i) Lagrangian and Hamiltonian descriptions of classical mechanics**
  - **(ii) relativistic mechanics; 4-momentum; relativistc free particle**

**(2) manifolds, tensors**
- **(a) manifolds, basic definitions**
- **(b) tangent vectors; curves on a manifold; diffeomorphisms**
- **(c) covectors/dual vectors/one-forms**
- **(d) tensors**
  - **(i) operations on tensors: contraction, outer product**
  - **(ii) transformation properties**
  - **(iii) example: metric tensor**
  - **(iv) example: faraday tensor, electromagnetic field**
  - **(v) tensor notation**

**(3) curvature**
- **(a) covariant derivative**
  - **(i) partial derivative transforms badly**
  - **(ii) formal correction of partial derivative**
  - **(iii) algebraic definition of covariant derivative**
  - **(iv) many covariant derivatives exist on a manifold; connection**
  - **(v) parallel transport**
  - **(vi) $\exists!$ metric compatible covariant derivative**
- **(b) Riemann curvature tensor**
  - **(i) commutator of covariant derivatives $\rightarrow$ curvature tensor**
  - **(ii) parallel transport around infinitesimal closed loop**
  - **(iii) connection determines curvature**
  - **(iv) algebraic symmetry properties of curvature tensor**
  - **(v) counting active indices**
  - **(vi) differential identity for curvature tensor; Bianchi identity**
  - **(vii) contractions; Ricci tensor; Ricci scalar; Einstein tensor**
- **(c) example calculations: $S^2$ in detail**
  - **(i) metric**
  - **(ii) metric compatible Christoffel coefficients; connection**

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- **(iii) connection determines curvature tensor**
- **(iv) Ricci tensor; Ricci scalar**

**(d) geodesics**

**(e) Riemannian normal coordinates at a point**

**(4) general relativity**

**(a) cartoon overview of Lagrangian derivation of GR**

**(b) structural assumptions and fundamental objects**

- **(i) spacetime: a Lorentz manifold $(M, g)$**
- **(ii) matter fields; local causality and energy positivity postulates**
- **(iii) Lagrangian density**

**(c) integration on a manifold; metric and chart induced volume elements**

**(d) tensor density**

**(e) Examples of Lagrangian field theories**

- **(i) $\mathbb{R}$-valued scalar field → Klein-Gordon equation**
- **(ii) no matter fields → Vacuum Einstein equation**
- **(iii) gravity + scalar field → Einstein-Klein-Gordon system**
- **(iv) electromagnetic field → Maxwell's equations**
- **(v) gravity + em field → Einstein-Maxwell system**
- **(vi) $\mathbb{C}$-valued scalar field; internal symmetry**
- **(vii) em field + $\mathbb{C}$-valued scalar field → Maxwell-Klein-Gordon system**
- **(viii) Sketch: gravity + $\mathbb{C}$-valued scalar field + em field → Einstein-Maxwell-Klein-Gordon system**
- **(ix) Sketch: Yang-Mills and other gauge fields**

**(5) exact solutions**

**(6) discussion of student papers**

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