INTRODUCTION TO COSMOLOGY AND THE COSMOLOGICAL CONSTANT

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Abstract. This paper is written for the course apm426S, taught by James Colliander in Spring 2002, University of Toronto. It gives an introduction to cosmology, concentrating on the effects of the cosmological constant Λ in the context of isotropic and homogeneous universe. Solutions to the Einstein's equation with and without Λ are carefully derived, analyzed and compared. Some current problems with Λ are briefly mentioned at the end.

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1. Introduction

Reader beware. This paper is neither a comprehensive introduction to cosmology nor to the problem of cosmological constant. It should be looked at as a reading guide to the first half of Wald's chapter on cosmology, quickly diverging to the discussion of cosmological constant once it is encountered. The author chose to explain all the necessary topics required for thorough appreciation of the effect of the cosmological constant that were not covered during lectures. That was done at the expense of not mentioning theoretical problems with having small Λ. In the opinion of the author it is not a great loss since a person who knows enough to appreciate those problems would not benefit form this paper anyway. Those readers are referred start to Weinberg's publication.

2. Framework

Before starting to solve the Einstein equation and trying to determine the fate of the universe, we should pause and try to think of some basic properties of the world we live in. This section contains some simplifying assumptions about the universe, their definitions and consequences. The reader should keep in mind that the ideas presented below are only one of the many possible ways to view the world we live in. So take every assumption with a grain of salt.

2.1. **Homogeneity and Isotropy.** From the time of Copernicus people accepted the idea that there is nothing special about our position in the universe. There also does not seem to be a "preferred direction" to look at when one is staring at the sky. What about the sun, the moon and other heavenly bodies that look so pretty? On the grand scheme of things, they don't really matter. To say something about the whole universe, we need to think BIG. Therefore, in this paper we shall not pay attention to objects smaller than a galaxy.

Since this is (supposed to be) a rigorous mathematical paper, let us name the observations above by fancy names and give them rigorous definitions. The property of space being the same everywhere will be called *homogeneity*. Wald defines a spacetime to be (spatially) homogeneous if there exists a one-parameter family of spacelike hypersurfaces Σ_t foliating the spacetime such that for each t and for any points $p, q \in \Sigma_t$, there exists an isometry of the spacetime metric which takes p into q.

Formalizing the idea that space looks the same in all directions is a little bit more tricky. The problem is that now we need to define how one should "look" at the space. For example, to a person sitting in a closed room, the air in the room might seem the same in all directions. However, a bullet passing through the same room will encounter much larger air density in the direction of its travel than in any other orthogonal direction. Therefore, we cannot demand our universe to look the same in all directions to all observers. The most we can ask for is *existence of one* observer that sees the universe the same (A little more thought will reveal that such an observer must be unique but this fact is not necessary for this paper).

Shamelessly copying from Wald again, we say that a spacetime looks the same in all directions, or, is *spatially isotropic at each point*, if there exist a congruence of timelike curves (i.e. observers), with tangent vectors denoted u^a, filling the spacetime and satisfying the following property: given any point p and any two unit "spatial" tangent vectors $s_1^a, s_2^a \in V_p$ (i.e. $s_i^a \perp u^a$), there exists an isometry of spacetime metric g_{ab} which leaves p and u^a fixed but rotates s_1^a into s_2^a. A slightly more intuitive way to think about isotropy is that it is impossible to construct a geometrically preferred tangent vector orthogonal to u^a.

Even though there are homogeneous surfaces which are nowhere isotropic and surfaces isotropic at one point but not homogeneous, the two concepts are related. It is not hard to prove that a homogeneous surface which is isotropic at one point is isotropic everywhere. Galaxy counts and measurements of background radiation show that the universe is highly isotropic in our neighbourhood. And a little Copernican humility will convince us of homogeneity of space. Therefore, we can assume our universe to be isotropic and homogeneous everywhere.

A small clarification is in order before we proceed. The notions of isotropy and homogeneity only apply to the space part of spacetime. We do *not* assume that the universe looks the same in the past as it does to the right.

2.2. **Derivation of Robertson-Walker Metric.** The assumptions of isotropy and homogeneity seem to be reasonable ones but as we will see shortly, they are quite severe in restricting the possibilities for the spacetime metric. The crucial fact to notice is that the tangent vectors u^a to observers in the definition of isotropy must be orthogonal to the surfaces of homogeneity Σ_t (if not, then we can construct a preferred direction on Σ_t using the non-orthogonal component of u^a, which would violate isotropy of Σ_t). Therefore, we can construct a metric of spacetime, g_{ab}, in terms of one-dimensional metric on worldlines of isotropic observers and a three-dimensional metric h_{ab} of Σ_t's:

(1) $$g_{ab} = -u_a u_b + h_{ab} \quad .$$

If we parametrize the world lines by proper time τ of the observers, the choice of minus sign in front of $u_a u_b$ becomes clear.

Now let us try to find the metric h_{ab}. We will do it in an indirect way. Consider the Riemann curvature tensor ${}^{(3)}R_{ab}{}^{cd}$ on the surface of homogeneity Σ_t (sidescript 3 will remind us that we are working on a three dimensional manifold, not the whole spacetime). Given a (0,2)-tensor, ${}^{(3)}R_{ab}{}^{cd}$ returns a map from (2,0)-tensors into $\mathbb{R}$. But this map can be identified with a (0,2)-tensor. Furthermore, ${}^{(3)}R_{ab}{}^{cd}$ is symmetric. Therefore, we can view ${}^{(3)}R_{ab}{}^{cd}$ as a linear map from the space W of two-forms into W.

Symmetry of ${}^{(3)}R_{ab}{}^{cd}$ and some theorem of linear algebra imply that there must exist an orthonormal basis consisting of eigenvectors of ${}^{(3)}R_{ab}{}^{cd}$. Isotropy of Σ_t makes the corresponding eigenvalues equal (otherwise can construct a preferred direction on Σ_t). Therefore, ${}^{(3)}R_{ab}{}^{cd}$ is a multiple of the identity map on W:

(2) $${}^{(3)}R_{ab}{}^{cd} = K\,\delta^a{}_b \wedge \delta^c{}_d = K\,\delta^a{}_{[b}\delta^c{}_{d]} \quad .$$

Homogeneity implies that K must be constant. By twice contracting ${}^{(3)}R_{ab}{}^{cd}$ we can show that Σ_t must be a surface of constant curvature:

$${}^{(3)}R_{\sigma\nu} = \sum_\beta {}^{(3)}R_{\sigma\beta\nu}{}^\beta = \sum_\beta \left(\sum_\alpha h_{\alpha\nu}\; {}^{(3)}R_{\sigma\beta}{}^{\alpha\beta} \right) = K \sum_{\alpha,\beta} h_{\alpha\nu}\delta^\sigma{}_{[\beta}\delta^\alpha{}_{\beta]} \quad .$$

Setting $\alpha = \beta$ and using the fact that $\delta^\sigma{}_{[\beta}\delta^\beta{}_{\beta]} = \delta^\sigma{}_\beta$, get

$${}^{(3)}R_{\sigma\nu} = K \sum_\beta h_{\beta\nu}\delta^\sigma{}_\beta = K h_{\beta\nu} \quad .$$

Therefore, the Ricci scalar is

$${}^{(3)}R = \sum_\sigma {}^{(3)}R_\sigma{}^\sigma = \sum_\sigma h^{\sigma\sigma}\; {}^{(3)}R_{\sigma\sigma} = K \sum_\sigma h^{\sigma\sigma} h_{\sigma\sigma} = 3K \quad .$$

We have just showed that the Ricci curvature is the same at every of Σ_t. Therefore, we have only three choices, up to an isomorphism, for Σ_t: $S^3, \mathbb{R}^3$, and hyperboloid. In suitable coordinates, we can express the metric for all these surfaces as follows:

(3) $${}^{(3)}\mathrm{d}s^2 = \mathrm{d}\psi^2 + (\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2) \cdot \begin{cases} \sin^2\psi & \text{if } {}^{(3)}R > 0 \\ \psi^2 & \text{if } {}^{(3)}R = 0 \\ \sinh^2\psi & \text{if } {}^{(3)}R < 0 \end{cases}$$

This can be put into a more convenient form

(4) $${}^{(3)}\mathrm{d}s^2 = \frac{\mathrm{d}r^2}{1 - kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2) \quad .$$

Here we can assume $k = +1, 0$, or -1. Using previous arguments, we can lift the metric of Σ_t into the metric of spacetime:

(5) $$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + a(\tau) \cdot \left(\frac{\mathrm{d}r^2}{1 - kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2) \right) \quad .$$

The expression above is called the *Robertson-Walker* metric. Using innocent-looking assumptions of isotropy and homogeneity we were able to reduce the question of metric determination to the problem of finding time evolution of the homogeneous surfaces and sign of their curvature (i.e. finding $a(\tau)$ and k).

2.3. **Perfect Fluid.** This is the last preparatory concept we need to understand. A *perfect fluid* can be defined as a fluid which looks isotropic in its rest frame. The concept of a fluid can be stretched a little by thinking of stars, fields, and even the whole universe as a fluid. Of course, we already started viewing the universe as a perfect fluid when we put the homogeneity and isotropy constrains on it. So it might seem as if this section is extraneous. However, remember that the Einstein's equation involves a connection between space curvature and energy-matter density. And nothing was said of energy-matter density yet.

Sean Carroll, in his Lecture Notes on General Relativity pp28–29, does an excellent job in deriving the energy-momentum tensor $T^{\mu\nu}$ of a perfect fluid:

$$T_{ab} = (\rho + p)u_a u_b + p\,\eta_{ab} \quad , \tag{6}$$

where ρ is fluid density, p is pressure and η_{ab} is the flat Minkowski metric with $-{+}{+}{+}$ signature. We will not duplicate Carroll's explanation here but will merely note that the formula above is the covariant generalization of

$$\begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} = (\rho + p)\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}(1\;0\;0\;0) + p\,\eta^a{}_b \quad .$$

3. Cosmology without Λ

We are finally ready to dive into the Einstein's equation and discover the fate of (spatially isotropic and homogeneous) universe.

$$G_{ab} = R_{ab} - \frac{1}{2}Rg_{ab} = 8\pi T_{ab} \tag{7}$$

First, we need to compute G_{ab} for the Robertson-Walker metric. Doing so for the flat case is not too hard. The general case, however, is quite formidable. So the author let a machine do the calculations. From the printout of *Mathematica* session (given as an appendix) we just take the Einstein tensor[1]

$$G_{\tau\tau} = 3\left(\frac{k}{a^2} + \frac{\dot{a}^2}{a^2}\right) \qquad \text{and} \qquad G_{**} = -2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - \frac{k}{a^2} \quad , \tag{8}$$

where G_{**} are the "spatial" components of G_{ab}. Equating them with $8\pi T_{ab}$, get

$$3\left(\frac{k}{a^2} + \frac{\dot{a}^2}{a^2}\right) = 8\pi\rho \qquad \text{and} \qquad -2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - \frac{k}{a^2} = 8\pi p \quad . \tag{9}$$

Using the first equation to reduce the second, and rearranging the first, get

$$3\frac{\ddot{a}}{a} = -4\pi(\rho + 3p) \quad , \tag{10}$$

$$3\frac{\dot{a}^2}{a^2} = 8\pi\rho - 3\frac{k}{a^2} \quad . \tag{11}$$

From these equations we can determine qualitative properties of $a(\tau)$ (we can also solve these equations explicitly but it is actually easier just working with these equations).

Note that if $\rho > 0$ and $p \geqslant 0$, (10) implies that $\ddot{a} < 0$! This means that a universe satisfying (7) cannot be static. It must always be either expanding or contracting (except one instant when $\dot{a} = 0$, which would represent the switch from expansion to contraction).

Another thing to notice is that if $\dot{a} > 0$, i.e. if the universe is expanding, then as we go backwards in time, it must be expanding at a faster and faster rate. Therefore, less than $\frac{a}{\dot{a}} = H^{-1}$ "units of time" ago, $a = 0$ (H is the *Hubble constant*, by the way). This is the good old BigBang.

For the next prediction we have to work harder. To begin with, let us change (11):

$$3\dot{a}^2 = 8\pi\rho a^2 - 3k$$

differentiating with respect to τ,

$$6\dot{a}\ddot{a} = 8\pi\dot{\rho}a^2 + 2 \cdot 8\pi\rho a\dot{a} \quad .$$

Using (10),

$$\dot{\rho}a^2 + 3a\dot{a}(\rho + p) = 0 \quad . \tag{12}$$

As was mentioned previously, a perfect fluid can be used to model a wide variety of substances. The type depends on the relationship between ρ and p. Ordinary matter on galactic scale is modelled by $\rho > 0$ and $p = 0$ [2] Another common "fluid" is radiation. It is given by the relationship $p = \rho/3$.

[1] If the reader actually looks at the computations, he will notice that the Einstein's tensor is not the same as below: the spatial components of G must be multiplied by the inverse of the metric to get the answer below. Wald does it without any explanation and the author could not come up with a good reason to do that. Since G's spatial components must be the same, Wald is right, and that is why the corrected version has been presented in the paper.

[2] This means that the particle in the fluid, or galaxies in the universe, are at rest with respect to each other. How well this model describes the universe is up to the reader to decide.

Therefore, assuming that the universe is matter-dominated, equation (12) becomes

$$0 = \dot{\rho}a^2 + 3a\dot{a}\rho$$

multiplying by a get

$$0 = \dot{\rho}a^3 + 3a^2\dot{a}\rho = \frac{\mathrm{d}}{\mathrm{d}\tau}(\rho a^3)$$

Therefore, ρa^3= constant. A similar calculation for radiation shows that ρa^4= constant. The first relationship is just conservation of mass. The latter one shows that radiation energy decays faster than universe expansion (extra power of a is attributed to the red-shift effect).

Now let us take a fresh look at (11). If $k > 0$ and the universe is expanding, then the first term on the right side is going to 0 faster than the second one (for both matter-filled and radiation-filled models). Therefore, there must be a time when the universe reaches its maximum size. And since no static solutions are allowed, it must start recontracting right away!

4. Cosmology with Λ

4.1. **Patching Einstein's Equation.**

The first observation we made from (10) was the simplest and the most profound one. After 80 years of Hubble's discovery of expanding universe, it hardly seems surprising to us. However, the theory of relativity and, in particular (10), were created *before* Hubble. The discovery of a dynamic universe could have been made years earlier if Einstein was slightly more open-minded (although who are we to judge Einstein?). Instead, Einstein tried to fix his original equation (7) by introducing a new term containing metric times a constant:

$$R_{ab} - \frac{1}{2}Rg_{ab} + \Lambda g_{ab} = 8\pi T_{ab} \quad . \tag{13}$$

He called Λ *the cosmological constant.* Before going into calculations to show how this "fixes" the problem of a dynamic universe, let us try to understand why the correction is not unreasonable.

Recall that equation (7) was the simplest covariant generalization of the Poisson's formula for the Newton potential

$$\Delta\Phi = 4\pi G\rho \quad . \tag{14}$$

Right side generalized nicely to T_{ab} but there was some fuss on what to do with the left side. The generalization had to have zero divergence (to satisfy energy conservation) and be constructed using only first two derivatives of the metric. $R_{ab} - \frac{1}{2}Rg_{ab}$ turned out to be the simplest answer. In a very long and probably unenlightening calculation, Lovelock showed that

$$R_{ab} - \frac{1}{2}Rg_{ab} + \Lambda g_{ab} \tag{15}$$

is the most general four-dimensional (0,2)-tensor that is divergence free and is constructed out of only first two derivatives of g_{ab}. So Einstein's patch to the equation actually made sense.

It should be noted that the Newtonian limit of (13) is *not* the Poisson's formula. It is instead

$$\Delta\Phi = 4\pi G\rho + \Lambda \quad . \tag{16}$$

So the value of Λ better be small.

4.2. **Static Universe.**

Now let us see how the Λ-term affected the dynamics of an evolving universe. Since equation (13) can be re-written as

$$R_{ab} - \frac{1}{2}Rg_{ab} = 8\pi T_{ab} - \Lambda g_{ab} \quad , \tag{17}$$

there is no need to recalculate G_{ab}. Using (8), we get

$$3\frac{\dot{a}^2}{a^2} = 8\pi\rho - 3\frac{k}{a^2} + \Lambda \tag{18}$$

$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = 8\pi p + \frac{k}{a^2} - \Lambda \tag{19}$$

(Note that we may assume $g_{ab} = diag(-1, 1, 1, 1)$ locally, by the Principle of Equivalence). Using some algebra, we get equations corresponding to (10) and (11),

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(3p + \rho) + \frac{\Lambda}{3} \quad , \tag{20}$$

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3} \quad . \tag{21}$$

If we want our universe to be static, we should let $\ddot{a} = \dot{a} = 0$. This gives

$$\frac{4\pi}{3}(3p + \rho) = \frac{\Lambda}{3} \quad , \tag{22}$$

$$\frac{k}{a^2} = \frac{8\pi}{3}\rho + \frac{\Lambda}{3} \quad . \tag{23}$$

Since $\rho > 0$ and $p \geqslant 0$, equation (22) implies that $\Lambda > 0$. But (23) then implies $k > 0$, so $k = +1$ since we assume curvature to be normalized.

If the universe is assumed to be matter-dominated, we may set $p = 0$. Then (22) becomes

$$\Lambda = 4\pi\rho \quad . \tag{24}$$

Plugging this into (23) with $k = 1$ get

$$\frac{1}{a^2} = \frac{8\pi}{3}\rho + \frac{4\pi}{3}\rho = 4\pi\rho \quad . \tag{25}$$

This gives a relationship between density and "radius" of the universe:

$$a(\rho) = \frac{1}{\sqrt{4\pi\rho}} \quad . \tag{26}$$

For radiation dominated case, get a similar formula:

$$a(\rho) = \frac{\sqrt{3}}{\sqrt{16\pi\rho}} \quad . \tag{27}$$

Einstein's static solution is known to be unstable. Let us verify this fact. Assume the density changes by a small amount: $\rho' = \rho + \Delta$. Then the corresponding change in a can be found using Taylor formula:

$$a' - a = \Delta \cdot \frac{\mathrm{d}}{\mathrm{d}\rho}a(\rho) = -\frac{1}{2}4\pi\Delta \cdot (4\pi\rho)^{-3/2} = -2\pi\Delta \cdot a^3 \quad . \tag{28}$$

This means that if the density is, say, decreases then the size will increase. Furthermore, the a^3 term not only magnifies the size of change but does so at an increasing rate! Even if the universe is static at some point in time, slightest perturbation of its density will either lead to a fast collapse or an infinite expansion.

Of course, the question arises of how the density of the universe might be perturbed, even if slightly. The universe is supposed to be an all-encompassing, and therefore, a perfectly isolated system. So it seems that there is nothing to that might throw it off balance. But this question should be left to the experts in quantum mechanics and theology, not relativity.

5. Λ as Vacuum Density

Depending on which side of (7) you put Λg_{ab}, there is a different way to interpret it. On the left side, it is a correction to the curvature tensor. On the right , it is a correction to the energy-momentum tensor with the "usual" curvature tensor. In the latter case, we get spacetime curvature even if there is no matter present in any form! But Λg_{ab} should still be interpreted as an energy-momentum tensor of something. So, what inhabits the universe when the universe is empty? Darkness, or vacuum, depending how scientific you want the answer to be.

Just by moving the Λ-term to the right side we discovered that vacuum might have some energy. Let us dig a little deeper. Vacuum is another example of a perfect fluid. Therefore, its energy-momentum tensor should be given by (6). I.e.,

$$-\Lambda g_{ab} = (\rho + p)u_a u_b + p\, g_{ab} \quad .$$

This shows that for vacuum, $p = -rho = -\Lambda/8\pi$. If we now go back to the static solution, we recall that $\Lambda > 0$. This means that vacuum must have a positive density and negative pressure. A pressure is a force of nearby particles of a fluid on each other. Therefore, in a static model, we have a universal *repulsive* force that counterbalances gravity and keeps the universe in a sensitive equilibrium (to better appreciate this idea, it is helpful to blaspheme by thinking of gravity as a mere force rather than curvature of spacetime).

5.1. **deSitter Model.** By now, we have made so much progress studying vacuum that we might as well see how an empty universe evolves. To do that, we just substitute $p = -\rho = -\Lambda/8\pi$ into (20) and (21), and see what comes out:

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(-2\rho) + \frac{8\pi}{3}\rho = \frac{16\pi}{3}\rho \ , \tag{29}$$

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}\rho - \frac{k}{a^2} + \frac{8\pi}{3}\rho = \frac{16\pi}{3}\rho - \frac{k}{a^2} \ . \tag{30}$$

Substituting former into latter, get

$$\frac{\dot{a}^2}{a^2} = \frac{\ddot{a}}{a} - \frac{k}{a^2} \tag{31}$$

or

$$k = a\ddot{a} - \dot{a}^2 \ . \tag{32}$$

This equation can be solved right away. I will do only one case, $k = -1$. The guessing method works wonderfully,

$$a(\tau) = A\sin(B\tau) \qquad \text{or} \qquad a(\tau) = A\sinh(B\tau) \ . \tag{33}$$

In the first case,

$$\dot{a}(\tau) = AB\cos(B\tau) \qquad \text{and} \qquad \ddot{a}(\tau) = -AB^2\sin(B\tau) \ . \tag{34}$$

Therefore,

$$a\ddot{a} - \dot{a}^2 = -(A^2B^2\sin^2(B\tau) + A^2B^2\cos^2(B\tau)) = -1 \ , \tag{35}$$

which gives $AB = \pm 1$. Now put $a(\tau)$ into (29):

$$-B^2 = \frac{\ddot{a}}{a} = \frac{16\pi}{3}\rho = \frac{2\Lambda}{3} \ . \tag{36}$$

This shows that $\Lambda < 0$ and $B = \pm\sqrt{-\frac{2\Lambda}{3}}$. So the first solution to (32) is

$$a(\tau) = \sqrt{-\frac{3}{2\Lambda}}\sin\left(\sqrt{-\frac{2\Lambda}{3}}\tau\right) \ . \tag{37}$$

The coefficients for the second one are obtained similarly, yielding $\Lambda > 0$ and

$$a(\tau) = \sqrt{\frac{3}{2\Lambda}}\sinh\left(\sqrt{\frac{2\Lambda}{3}}\tau\right) \ . \tag{38}$$

So a negatively curved space with negative vacuum density will recollapse but the one with positive density will expand forever. This confirms out idea of viewing positive vacuum energy as a repulsive force. The second solution, and the solutions to the cases $k = 0, +1$, are called the *De Sitter* model (They are supposed to be isomorphic and maximally symmetric, i.e. empty universe *does* look the same to the past as it does to the right. But of course, there is nobody to appreciate the view.) The first one is anti-de Sitter model.

6. Issues with Λ

If Λ was introduced to produce static solutions to (7) and we know that our universe is not static, why worry about Λ? Because it is a valid term in the Einstein's equation and it may actually contribute to the evolution of our universe. The only way to get rid of Λ is to measure both sides of (7) and show that Λ is negligible. Numerous measurements have been done, primarily using red-shift effect. And the answer is indeed small, although still far from 0. The problem is that when people try to *calculate* Λ using other theories of physics, primarily quantum mechanics, they must explain how a number of seemingly independent quantities thirty orders larger in magnitude cancel each other out just right as to yield a very small value of Λ. Furthermore, Λ seems to be a quick fix for other cosmological problems that looks more and more appealing with every year. As a matter of fact, the current ideas about our universe need a significant contribution from Λ to explain the way things are now. So Einstein may not have blundered after all when he introduced the cosmological constant.

7. Bibliography

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