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In[1]:= (* v contains variable names of the chart *)
v = {t, r,  $\theta$ ,  $\phi$ };
(* g is the metric  $g_{\mu\nu}$  *)
g = DiagonalMatrix[{-1,  $\frac{a[t]^2}{1 - k r^2}$ ,  $a[t]^2 r^2$ ,  $a[t]^2 r^2 \sin[\theta]^2$ }] ;
(* gInv is the inverse metric  $g^{\mu\nu}$  *)
gInv = Inverse[g];

In[2]:= (* Calculating  $\Gamma^\rho_{\mu\nu} = \Gamma[\rho, \mu, \nu]$  by formular (3.1.30) of Wald *)
 $\Gamma[\rho\_ , \mu\_ , \nu\_ ] := \Gamma[\rho, \mu, \nu] =$ 
FullSimplify[ $\frac{1}{2} \sum_{\sigma=1}^4 gInv[[\rho, \sigma]] (\partial_{v[[\mu]]} g[[\nu, \sigma]] + \partial_{v[[\nu]]} g[[\mu, \sigma]] - \partial_{v[[\sigma]]} g[[\mu, \nu]])$ ];

(* Calculating the Ricci tensor  $R_{\mu\rho} = R[\mu, \rho]$  by (3.4.5) of Wald *)
 $R[\mu\_ , \rho\_ ] := R[\mu, \rho] =$  FullSimplify[ $\sum_{\nu=1}^4 \partial_{v[[\nu]]} \Gamma[\nu, \mu, \rho] -$ 
 $\partial_{v[[\mu]]} \left( \sum_{\nu=1}^4 \Gamma[\nu, \nu, \rho] \right) + \sum_{\alpha=1}^4 \left( \sum_{\nu=1}^4 (\Gamma[\alpha, \mu, \rho] \Gamma[\nu, \alpha, \nu] - \Gamma[\alpha, \nu, \rho] \Gamma[\nu, \alpha, \mu]) \right)$ ];

(* Calculate the Ricci scalar  $RR = R_{\mu\nu} g^{\mu\nu}$  *)
RR = FullSimplify[ $\sum_{\mu=1}^4 \left( \sum_{\nu=1}^4 R[\mu, \nu] gInv[[\nu, \mu]] \right)$ ];

(* Finally, calculate  $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} RR g_{\mu\nu}$  *)
 $G[\mu\_ , \nu\_ ] := G[\mu, \nu] =$  FullSimplify[ $R[\mu, \nu] - \frac{1}{2} RR g[[\mu, \nu]]$ ];

In[3]:= (* Display all the  $\Gamma$ 's, one table for each upper index, starting with v[[1]] *)

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In[4]:= Table[MatrixForm[Table[R[m, i, j], {i, 1, 4}, {j, 1, 4}], {m, 1, 4}] // TableForm
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Out[4]//TableForm=
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$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{a[t] a'[t]}{1-k r^2} & 0 & 0 \\ 0 & 0 & r^2 a[t] a'[t] & 0 \\ 0 & 0 & 0 & r^2 a[t] \sin[\theta]^2 a'[t] \end{pmatrix}$$

$$\begin{pmatrix} 0 & \frac{a'[t]}{a[t]} & 0 & 0 \\ \frac{a'[t]}{a[t]} & \frac{k r}{1-k r^2} & 0 & 0 \\ 0 & 0 & r (-1+k r^2) & 0 \\ 0 & 0 & 0 & r (-1+k r^2) \sin[\theta]^2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & \frac{a'[t]}{a[t]} & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ \frac{a'[t]}{a[t]} & \frac{1}{r} & 0 & 0 \\ 0 & 0 & 0 & -\cos[\theta] \sin[\theta] \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & \frac{a'[t]}{a[t]} \\ 0 & 0 & 0 & \frac{1}{r} \\ 0 & 0 & 0 & \cot[\theta] \\ \frac{a'[t]}{a[t]} & \frac{1}{r} & \cot[\theta] & 0 \end{pmatrix}$$

(* Display Ricci tensor as a 4*4 matrix *)

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In[5]:= Table[R[i, j], {i, 1, 4}, {j, 1, 4}] // MatrixForm
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Out[5]//MatrixForm=
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$$\begin{pmatrix} -\frac{3 a''[t]}{a[t]} & 0 & 0 & 0 \\ 0 & \frac{2 (k+a'[t]^2)+a[t] a''[t]}{1-k r^2} & 0 & 0 \\ 0 & 0 & r^2 (2 (k+a'[t]^2)+a[t] a''[t]) & 0 \\ 0 & 0 & 0 & r^2 \sin[\theta]^2 (2 (k+a'[t]^2)+a[t] a''[t]) \end{pmatrix}$$

(* Ricci curvature *)

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In[6]:= RR
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$$\text{Out[6]} = \frac{6 (k+a'[t]^2)+a[t] a''[t]}{a[t]^2}$$

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In[7]:= (* And finally, Einstein's tensor *)
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Table[G[i, j], {i, 1, 4}, {j, 1, 4}] // MatrixForm
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Out[7]//MatrixForm=
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$$\begin{pmatrix} \frac{3 (k+a'[t]^2)}{a[t]^2} & 0 & 0 & 0 \\ 0 & \frac{k+a'[t]^2+2 a[t] a''[t]}{-1+k r^2} & 0 & 0 \\ 0 & 0 & -r^2 (k+a'[t]^2+2 a[t] a''[t]) & 0 \\ 0 & 0 & 0 & -r^2 \sin[\theta]^2 (k+a'[t]^2+2 a[t] a''[t]) \end{pmatrix}$$