On generalization of Odd Khovanov Homologies

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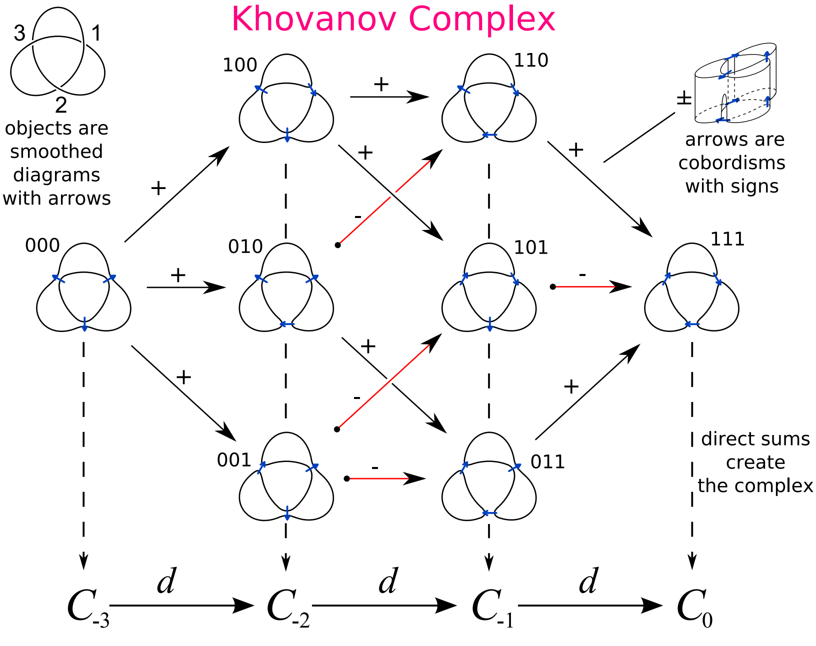
Khovanov Complex

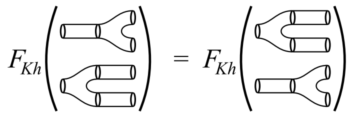


Khovanov functor

see Khovanov: arXiv:math/9908171

F_{Kh}: Cob $\longrightarrow$ $\mathbb{Z}$-Mod

"symmetric":



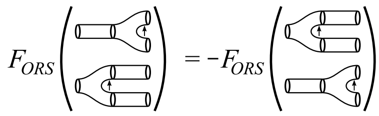
Edge assignment given explicite.

ORS "half-projective" functor

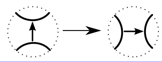
see Ozsvath, Rasmussen and Szabo: arXiv:0710.4300

F_{ORS}: ArCob $\longrightarrow$ $\mathbb{Z}$-Mod

not "symmetric":



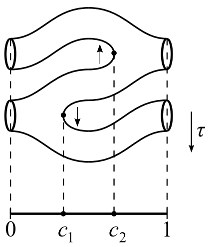
Edge assignment given by homological properties.

Arrows in smoothed diagrams: 

Problem Can **Cob** be changed to make F_{ORS} a functor?

Motivation Invariance of the complex may be proved on the level of topology.

ChCob: cobordisms with chronology & arrows

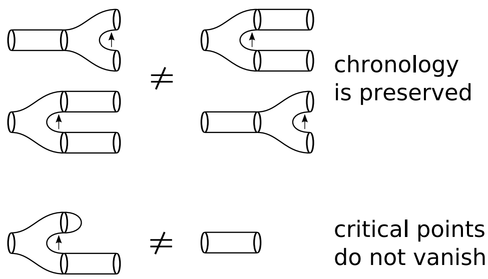


Chronology τ is a Morse function with exactly one critical point over each critical value.

Critical points of index 1 have arrows.

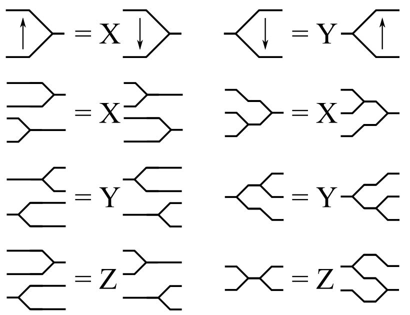
Chronology isotopy: smooth homotopy H satisfying:

- $H_0 = \tau_0$
- $H_1 = \tau_1$
- H_t is a chronology



Which conditions should F: **ChCob** $\longrightarrow$ $\mathbb{Z}$**-Mod** satisfy to produce homologies?

Chronology change condition



$$X^2 = Y^2 = Z^2 = 1$$

This is to have well defined coefficient for any permutation of critical points (change of chronology).

Four solutions over $\mathbb{Z}$:

(X,Y,Z) = (±1,±1,±1)/±1

Khovanov: (1, 1, 1)

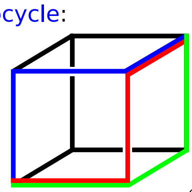
ORS: (1, -1, 1)

Ch.ch.c. gives egde assignment!

Each square S corresponds to change of chronology with some coefficient λ. The cochain

$$\psi(S) = -\lambda$$

is a cocycle:



$$P = \lambda_r P = \lambda_r \lambda_f P = \ldots = \prod_{i=1}^{6} \lambda_i P$$

By ch.ch.c.:

$$d\psi(C) = \prod_{i=1}^{6}(-\lambda_i) = 1$$

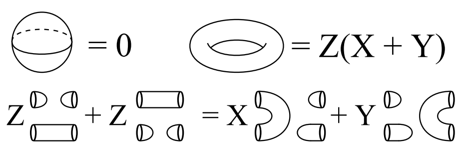
and by contractibility:

$$\psi = d\varphi$$

φ is the edge assignment.

S/T/4Tu relations

compare with Bar-Natan: arXiv:math/0410495



- all arrows point upwards
- no other critical points between

Does chronology help?

Theorem 1 The complex is a link invariant upto homotopy, ch.ch.c and S/T/4Tu.

Theorem 2 There exists a functor

F_U : **ChCob** $\longrightarrow$ R-**Mod**

satisfying ch.ch.c and S/T/4Tu, where

R = $\mathbb{Z}$[X,Y,Z]/(X^2=Y^2=Z^2=1)

(X,Y,Z) = (1,1,1):

F_U is the Khovanov functor with c=0

(X,Y,Z) = (1,-1,1):

F_U is the ORS functor