Dror Bar-Natan — Research Plan

(web version: http://www.math.toronto.edu/~drorbn/ResearchPlan/)
last updated: Oct. 19, 2002

The phrase "research plan" is almost an oxymoron, for if it can be planned, it ain't research. Or at least, if it can be planned it's already half done, and hence it isn't the best of research. Thus my primary plan of mathematical research for the next few years is to **follow my nose**. The scents that usually attract me most are

- *Structure.* Some like riddles or hard problems or the very applied end. I've done some riddles (e.g. [3, 5]) and problems (e.g. [4, 10]) and even played with politically charged mathematics (e.g. [9]). But what I have always enjoyed most is the realization that there is more to a subject than the list of propositions it is formally made of, and the subjects I liked most are those in which structure plays a dominant role and in which much of the content becomes obvious once one holds the right philosophy. I tend to like the elementary, which can often be explained to a general mathematical audience within an hour or two (e.g. [1, 6, 8, 11]), but when the reasons were good enough I've enjoyed depth (e.g. [2, 7]).

  Inertia shows that I'm likely to look for (algebraic) structure in the vicinity of knot theory and low dimensional topology from the perspective of quantum algebra and topological quantum field theory. *Aside:* The well established fact that this is interesting remains a surprise for me. That knot theory is topologically interesting is no surprise at all. But the depth and the beauty of the theory of combinatorial/algebraic/quantum knot invariants goes way beyond one's naive expectations. What a lucky coincidence!

- *Pictorial simplicity.* I often think in pictures and I like to tell my stories in pictures (see e.g. [1]–[13]). I will go on in the same way.

- *Computer assisted mathematics.* I love to program and I've often embedded programs and computer assisted calculations in my papers (e.g. [2, 11]).

- *Weblications.* I like to post little droplets of mathematics (often but not always very elementary) on the web, whether or not they are related to my main stream of research (see them at http://www.math.toronto.edu/~drorbn/).

My backup plans are (in case I become anosmic, or if today's strong smells remain dominant in the future):

- I feel there is a great need for a foundational book on finite type invariants that will summarize the relationship between those and Lie algebras, associators, Feynman diagrams and the Chern-Simons theory, etc. I've been meaning to write such a book, but I still find that even the foundations of the theory are not yet fully settled, and hence a pre-condition for writing it is to do some foundational research. Just as an example — along with D. P. Thurston I'm trying to rewrite Drinfel'd's theory of associators and the classical theory of *6j* symbols in terms of knotted trivalent graphs, tetrahedra and Pachner moves. Much is already done but much remains to be done. See e.g. [12]. Much foundational work also remains regarding Feynman diagrams and configuration space integrals. And then there's writing the book.

- I plan to continue following Khovanov's novel "categorification" (see [11]).

References

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[8] (With S. Garoufalidis, L. Rozansky and D. P. Thurston) *The Århus integral of rational homology 3-spheres I: A highly non trivial flat connection on* S^3, Selecta Mathematica, to appear.

[9] (With M. Bar-Hillel, G. Kalai and B. D. McKay) *Solving the Bible Code Puzzle,* Statistical Science **14-2** (1999) 150–173.

[10] (With R. Lawrence) *A Rational Surgery Formula for the LMO Invariant,* Israel Journal of Mathematics, to appear.

[11] *On Khovanov's categorification of the Jones polynomial,* http://www.math.toronto.edu/~drorbn/papers/Categorification/.

[12] (With D. P. Thurston) *Algebraic Structures on Knotted Objects and Universal Finite Type Invariants,* http://www.math.toronto.edu/~drorbn/papers/AlgebraicStructures/.

[13] Other papers by Dror Bar-Natan, see http://www.math.toronto.edu/~drorbn/LOP.html