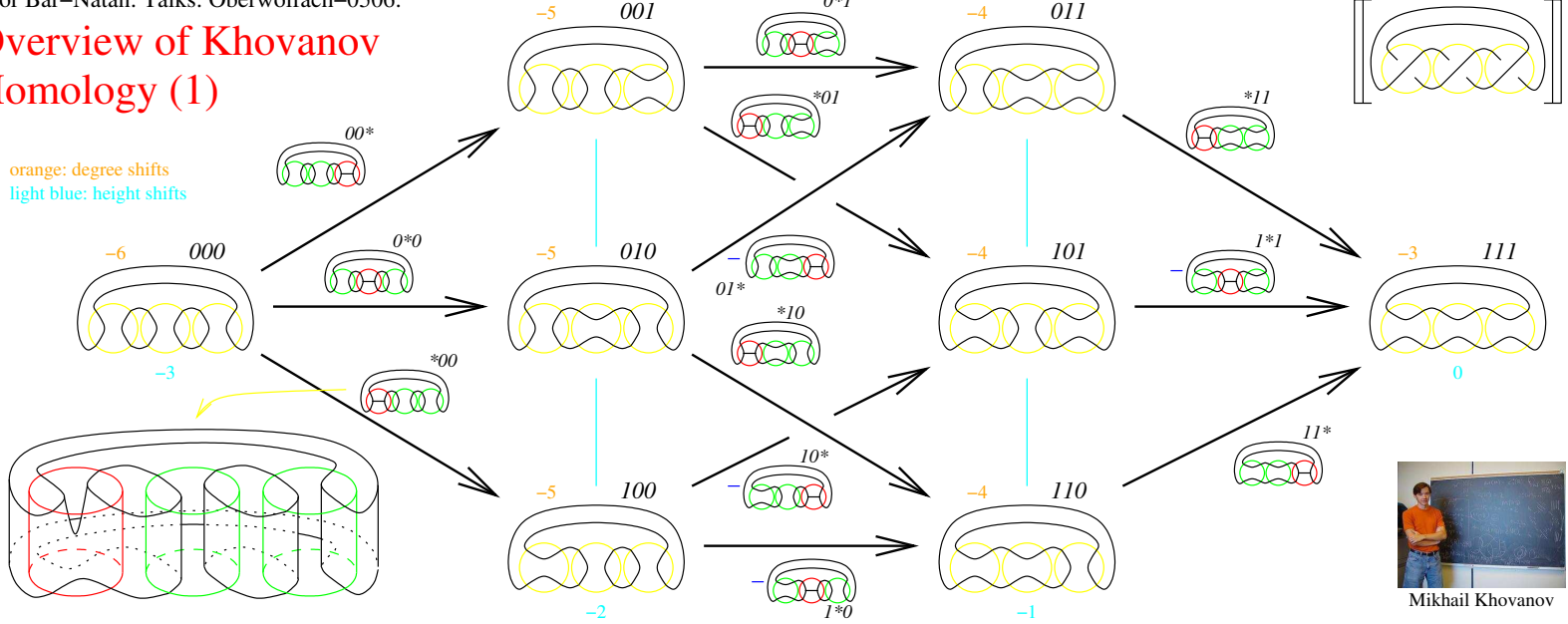


Overview of Khovanov Homology (1)

orange: degree shifts
light blue: height shifts

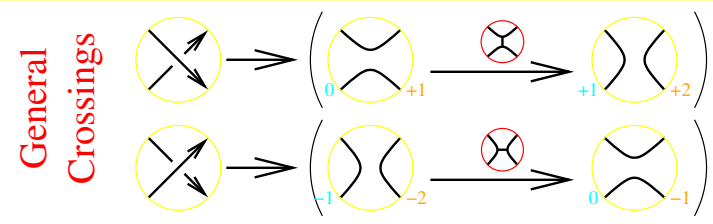
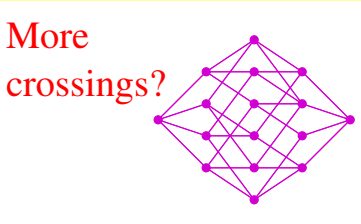
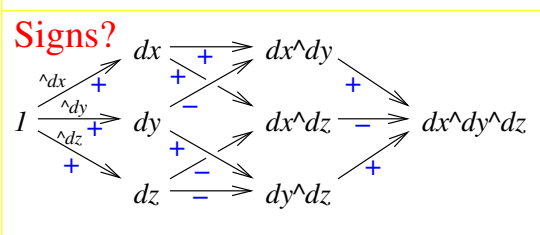


What is it? A cube for each knot/link projection;

Vertices: All fillings of with or with .

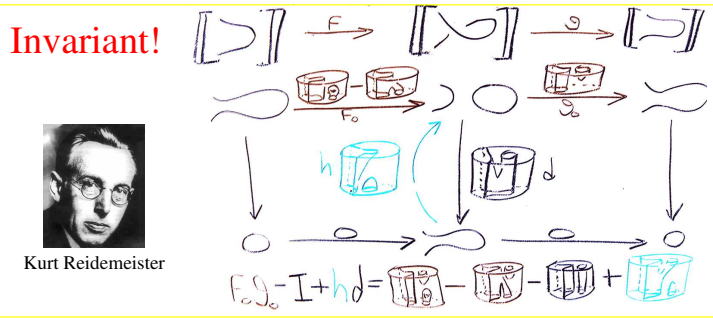
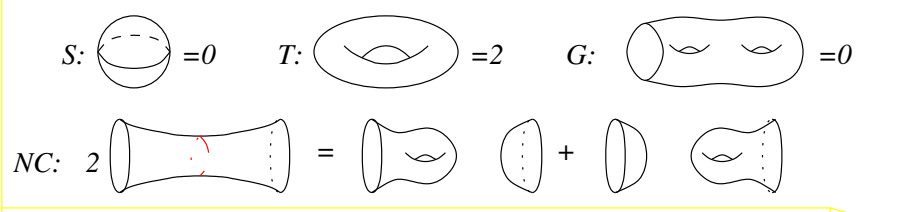
Edges: All fillings of $I \times$ = with $I \times$ = or with $I \times$ = and precisely one .

"God created the knots, all else in topology is the work of mortals"
Leopold Kronecker (modified)

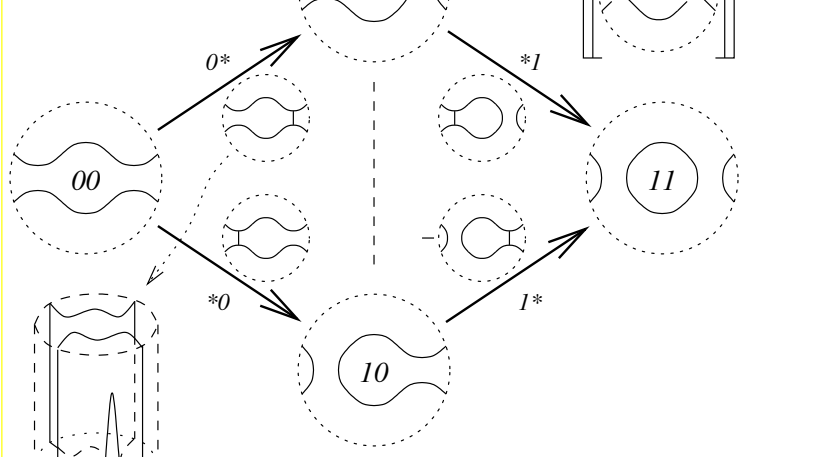


Where does it live? In $Kom(Mat(\langle Cob \rangle / \{S, T, G, NC\})) / \text{homotopy}$

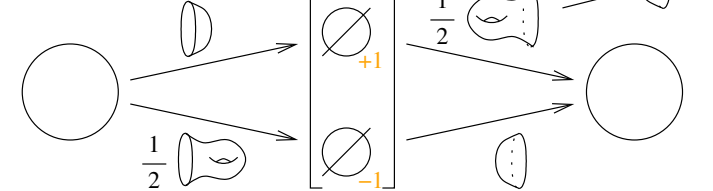
Kom: Complexes *Mat*: Matrices *Cob*: Cobordisms $\langle \dots \rangle$: Formal lin. comb.



The case of tangles:



Computable! via "complex simplification":



The Reduction Lemma. If ϕ is an isomorphism then the complex

$$[C] \xrightarrow{\begin{pmatrix} \alpha \\ \beta \end{pmatrix}} \begin{bmatrix} b_1 \\ D \end{bmatrix} \xrightarrow{\begin{pmatrix} \phi & \delta \\ \gamma & \epsilon \end{pmatrix}} \begin{bmatrix} b_2 \\ E \end{bmatrix} \xrightarrow{(\mu \ \nu)} [F]$$

is isomorphic to the (direct sum) complex

$$[C] \xrightarrow{\begin{pmatrix} 0 \\ \beta \end{pmatrix}} \begin{bmatrix} b_1 \\ D \end{bmatrix} \xrightarrow{\begin{pmatrix} \phi & 0 \\ 0 & \epsilon - \gamma\phi^{-1}\delta \end{pmatrix}} \begin{bmatrix} b_2 \\ E \end{bmatrix} \xrightarrow{(0 \ \nu)} [F]$$