Preliminaries

This is IType.nb of ωεβ/ap.

```
Once[<< KnotTheory`; << Rot.m];
```

```
Loading KnotTheory` version
    of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
Loading Rot.m from
    http://drorbn.net/AP/Talks/Groningen-240530
    to compute rotation numbers.
```

```
CF[ω_. ℰ_𝔼] := CF[ω] × CF /@ ℰ;
CF[ℰ_List] := CF /@ ℰ;
CF[ℰ_] := Module[{vs, ps, c},
   vs = Cases[ℰ, (x | p | ξ | π)__, ∞] ∪ {x, p, ϵ};
   Total[CoefficientRules[Expand[ℰ], vs] /.
     (ps_ → c_) :> Factor[c] (Times @@ vs^ps)]];
```

Integration

Using Picard Iteration!

```
𝔼 /: 𝔼[A_] × 𝔼[B_] := 𝔼[A + B];
```

```
$π = Identity; (* hacks in pink *)
```

```
Unprotect[Integrate]; (* keys in yellow *)
∫ω_. 𝔼[L_] 𝔡(vs_List) :=
  Module[{n, L0, Q, Δ, G, Z0, Z, λ, DZ, FZ, a, b},
   n = Length@vs; L0 = L /. ϵ → 0;
   Q = Table[(-∂vs⟦a⟧,vs⟦b⟧ L0) /. Thread[vs → 0] /.
      (p | x)__ → 0, {a, n}, {b, n}];
   If[(Δ = Det[Q]) == 0, Return@"Degenerate Q!"];
   Z = Z0 = CF@$π[L + vs.Q.vs / 2]; G = Inverse[Q];
   DZa_ := ∂vs⟦a⟧ Z; DZa_,b_ := ∂vs⟦b⟧ DZa;
   FZ := CF@$π[1/2 Sum[G⟦a, b⟧ (DZa,b + DZa DZb), {a, n}, {b, n}]];
   FixedPoint[(Z = Z0 + ∫0^λ FZ 𝔡λ) &, Z];
   PowerExpand@Factor[ω Δ^-1/2] ×
    𝔼[CF[Z /. λ → 1 /. Thread[vs → 0]]]];
Protect[Integrate];
```

```
∫𝔼[-μ x^2/2 + i ξ x] 𝔡{x}
```

$$\frac{\mathbb{E}\left[-\frac{\xi^2}{2\mu}\right]}{\sqrt{\mu}}$$

```
L = -1/2 {x1, x2}.{{a, b}, {b, c}}.{x1, x2} + {ξ1, ξ2}.{x1, x2};
Z12 = ∫𝔼[L] 𝔡{x1, x2}
```

$$\frac{\mathbb{E}\left[\frac{c\,\xi_1^2}{2\,(-b^2+a\,c)} + \frac{b\,\xi_1\,\xi_2}{b^2-a\,c} + \frac{a\,\xi_2^2}{2\,(-b^2+a\,c)}\right]}{\sqrt{-b^2+a\,c}}$$

```
{Z1 = ∫𝔼[L] 𝔡{x1}, Z12 == ∫Z1 𝔡{x2}}
```

$$\left\{\frac{\mathbb{E}\left[-\frac{(-b^2+a\,c)\,x_2^2}{2\,a} - \frac{b\,x_2\,\xi_1}{a} + \frac{\xi_1^2}{2\,a} + x_2\,\xi_2\right]}{\sqrt{a}}, \text{True}\right\}$$

```
$π = Normal[# + O[ϵ]^13] &; ∫𝔼[-ϕ^2/2 + ϵ ϕ^3/6] 𝔡{ϕ}
```

$$\mathbb{E}\left[\frac{5\,\epsilon^2}{24} + \frac{5\,\epsilon^4}{16} + \frac{1105\,\epsilon^6}{1152} + \frac{565\,\epsilon^8}{128} + \frac{82\,825\,\epsilon^{10}}{3072} + \frac{19\,675\,\epsilon^{12}}{96}\right]$$

From https://oeis.org/A226260:

THE ON-LINE ENCYCLOPEDIA OF INTEGER SEQUENCES®

founded in 1964 by N. J. A. Sloane

Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

A226260 Numerators of mass formula for connected vacuum graphs on 2n nodes for a phi^3 field theory.

1, 5, 5, 1105, 565, 82825, 19675, 1282031525, 80727925, 1683480621875, 13209845125, 2239646759308375, 19739117098375, 6320791709083309375, 32468078556378125, 3836267676845045751875, 2813657784050329731 25, 2824650747089425586152484375, 77663215703411671273 4375 (list; graph; refs; listen; history; text; internal format)

The Right-Handed Trefoil

```
K = Mirror@Knot[3, 1]; Features[K]
```

```
Features[7, C4[-1] X1,5[1] X3,7[1] X6,2[1]]
```

```
ℒ[Xi_,j_[s_]] := T^s/2 𝔼[
   xi (pi+1 - pi) + xj (pj+1 - pj) +
    (T^s - 1) xi (pi+1 - pj+1) +
    (ϵ s / 2) ×
     (xi (pi - pj) ((T^s - 1) xi pj + 2 (1 - xj pj)) - 1)]
ℒ[Ci_[φ_]] := T^φ/2 𝔼[xi (pi+1 - pi) + ϵ φ (1/2 - xi pi)]
ℒ[K_] := CF[ℒ /@ Features[K]⟦2⟧]
vs[K_] :=
 Join @@ Table[{pi, xi}, {i, Features[K]⟦1⟧}]
```

```
{vs[K], ℒ[K]}
```

$$\Big\{\{p_1, x_1, p_2, x_2, p_3, x_3, p_4, x_4, p_5, x_5, p_6, x_6, p_7, x_7\},$$
$$T\,\mathbb{E}\Big[-2\,\epsilon - p_1\,x_1 + \epsilon\,p_1\,x_1 + T\,p_2\,x_1 - \epsilon\,p_5\,x_1 + (1-T)\,p_6\,x_1 +$$
$$\frac{1}{2}(-1+T)\,\epsilon\,p_1\,p_5\,x_1^2 + \frac{1}{2}(1-T)\,\epsilon\,p_5^2\,x_1^2 - p_2\,x_2 +$$
$$p_3\,x_2 - p_3\,x_3 + \epsilon\,p_3\,x_3 + T\,p_4\,x_3 - \epsilon\,p_7\,x_3 + (1-T)\,p_8\,x_3 +$$
$$\frac{1}{2}(-1+T)\,\epsilon\,p_3\,p_7\,x_3^2 + \frac{1}{2}(1-T)\,\epsilon\,p_7^2\,x_3^2 - p_4\,x_4 +$$
$$\epsilon\,p_4\,x_4 + p_5\,x_4 - p_5\,x_5 + p_6\,x_5 - \epsilon\,p_1\,p_5\,x_1\,x_5 +$$
$$\epsilon\,p_5^2\,x_1\,x_5 - \epsilon\,p_2\,x_6 + (1-T)\,p_3\,x_6 - p_6\,x_6 +$$
$$\epsilon\,p_6\,x_6 + T\,p_7\,x_6 + \epsilon\,p_2^2\,x_2\,x_6 - \epsilon\,p_2\,p_6\,x_2\,x_6 +$$
$$\frac{1}{2}(1-T)\,\epsilon\,p_2^2\,x_6^2 + \frac{1}{2}(-1+T)\,\epsilon\,p_2\,p_6\,x_6^2 -$$
$$p_7\,x_7 + p_8\,x_7 - \epsilon\,p_3\,p_7\,x_3\,x_7 + \epsilon\,p_7^2\,x_3\,x_7\Big]\Big\}$$

```
$π = Normal[# + O[ϵ]^2] &; ∫ℒ[K] 𝔡(vs@K)
```

$$-\frac{i\,T\,\mathbb{E}\left[-\frac{(-1+T)^2\,(1+T^2)\,\epsilon}{(1-T+T^2)^2}\right]}{1-T+T^2}$$

Video and more at http://www.math.toronto.edu/~drorbn/Talks/Groningen-240530.