

Implementation (sources: <http://drorbn.net/icerm23/ap>). I like it most when the implementation matches the math perfectly. We failed here.

Once[<< KnotTheory`];

Loading KnotTheory` version

of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

Utilities. The step function, algebraic numbers, canonical forms.

$\theta[x_]$ /; NumericQ[x_] := UnitStep[x]

```
 $\omega 2[v\_][p\_]$  := Module[{q = Expand[p], n, c},
  If[q ===  $\theta$ ,  $\theta$ ,
    c = Coefficient[q,  $\omega$ , n = Exponent[q,  $\omega$ ]];
    c v^n +  $\omega 2[v][q - c (\omega + \omega^{-1})^n]$ ];
```

```
sign[ $\mathcal{E}$ _] := Module[{n, d, v, p, rs, e, k},
  {n, d} = NumeratorDenominator[ $\mathcal{E}$ ];
  {n, d} /=  $\omega^{\text{Exponent}[n, \omega] / 2 + \text{Exponent}[n, \omega, \text{Min}] / 2}$ ;
  p = Factor[ $\omega 2[v]@n * \omega 2[v]@d /. v \rightarrow 4 u^2 - 2$ ];
  rs = Solve[p ==  $\theta$ , u, Reals];
  If[rs === {}, Sign[p /. u  $\rightarrow \theta$ ],
    rs = Union@{u /. rs};
    Sign[(-1)^{e=Exponent[p, u]} Coefficient[p, u, e]] + Sum[
      k =  $\theta$ ;
      While[{d = RootReduce[ $\partial_{\{u, ++k\}} p /. u \rightarrow r$ ]} ==  $\theta$ ];
      If[EvenQ[k],  $\theta$ , 2 Sign[d]] *  $\theta[u - r]$ ,
      {r, rs}]
  ]
]
```

SetAttributes[B, Orderless];

CF[b_B] := RotateLeft[#, First@Ordering[#] - 1] & /@ DeleteCases[b, {}]

CF[\mathcal{E} _] := Module[{ γs = Union@Cases[\mathcal{E} , γ | $\bar{\gamma}$, ∞]},
 Total[CoefficientRules[\mathcal{E} , γs] /.
 (ps_ \rightarrow c_) \Rightarrow Factor[c] \times Times@@ γs^{ps}]

CF[{}] = {};

CF[C_List] :=

```
Module[{ $\gamma s$  = Union@Cases[C,  $\gamma$ ,  $\infty$ ],  $\gamma$ },
  CF /@ DeleteCases[ $\theta$ ] [
    RowReduce[Table[ $\partial_{\gamma} r$ , {r, C}, { $\gamma$ ,  $\gamma s$ }]]. $\gamma s$  ]
```

(\mathcal{E})^* := $\mathcal{E} /. \{\bar{\gamma} \rightarrow \gamma, \gamma \rightarrow \bar{\gamma}, \omega \rightarrow \omega^{-1}, c_Complex \Rightarrow c^*\}$;

r_Rule^* := {r, r^*}

RulesOf[$\gamma_i + rest_.$] := ($\gamma_i \rightarrow -rest$)^+;

CF[PQ[C_, q_]] := Module[{nc = CF[C]},
 PQ[nc, CF[q /. Union@@RulesOf /@ nc]]]

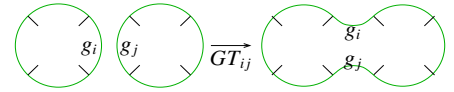
CF[$\Sigma_b[\sigma_, pq_]$] := $\Sigma_{CF[b]}[\sigma, CF[pq]]$

Pretty-Printing.

```
Format[ $\Sigma_{b_B}[\sigma_, PQ[C_, q\_]]$ ] := Module[{ $\gamma s$ },
   $\gamma s$  =  $\gamma\#$  & /@ Join@@b;
  Column[{TraditionalForm@ $\sigma$ ,
    TableForm[Join[
      Prepend[""] /@ Table[TraditionalForm[ $\partial_{c,r}$ ],
        {r, C}, {c,  $\gamma s$ }],
      {Prepend[""] [
        Join@@
          (b /. {L_, m___, r_}  $\Rightarrow$ 
            {DisplayForm@RowBox[{"(", L}],
              m, DisplayForm@RowBox[{r, ")"}]}) /.
            i_Integer  $\Rightarrow \gamma_i$  ]},
      MapThread[Prepend,
        {Table[TraditionalForm[ $\partial_{r,c} q$ ], {r,  $\gamma s^*$ },
          {c,  $\gamma s$ }],  $\gamma s^*$ }
      ], TableAlignments  $\rightarrow$  Center]
    ], Center] ];
```

The Face-Centric Core.

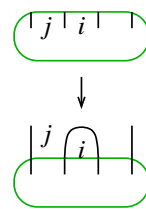
$\Sigma_{b1}[\sigma_1, PQ[C1_, q1_]] \oplus \Sigma_{b2}[\sigma_2, PQ[C2_, q2_]] \wedge :=$
 CF@ $\Sigma_{\text{Join}[b1, b2]}[\sigma_1 + \sigma_2, PQ[C1 \cup C2, q1 + q2]]$;



GT for Gap Touch:

GT_{i,j}@ $\Sigma_B[\{li_, i, ri_ \}, \{lj_, j, rj_ \}, bs_]$ [σ ,
 PQ[C_, q_]] :=

CF@ $\Sigma_B[\{ri, li, j, rj, lj, i, bs\}] [\sigma, PQ[C \cup \{\gamma_i - \gamma_j\}, q]]$



cor·don (kōr'dn)

n.

1. A line of people, military posts, or ships stationed around an area to enclose or guard it: *a police cordon*.
2. A rope, line, tape, or similar border stretched around an area, usually by the police, indicating that access is restricted.



use ϕ_p to kill its row and column, drop a $\begin{pmatrix} 01 \\ 10 \end{pmatrix}$ summand

$$s \begin{pmatrix} 0 & \phi C_{rest} \\ \bar{\phi}^T & \lambda \theta \\ \bar{C}_{rest}^T & \bar{\theta}^T A_{rest} \end{pmatrix} \rightarrow \begin{cases} \exists p \phi_p \neq 0 & \text{use } \phi_p \text{ to kill its row and column, drop a } \begin{pmatrix} 01 \\ 10 \end{pmatrix} \text{ summand} \\ \phi = 0, \lambda \neq 0 & \text{use } \lambda \text{ to kill } \theta, \text{ let } s += \text{sign}(\lambda) \\ \phi = 0, \lambda = 0 & \text{append } \theta \text{ to } C_{rest}. \end{cases}$$

Cordon_i@ $\Sigma_B[\{li_, i, ri_ \}, bs_]$ [σ , PQ[C_, q_]] :=

```
Module[{ $\phi = \partial_{\gamma_i} C$ ,  $\lambda = \partial_{\bar{\gamma}_i} q$ ,  $n\sigma = \sigma$ ,  $nC$ ,  $nq$ , p},
  {p} = FirstPosition[({# !=  $\theta$ ) & /@  $\phi$ , True, { $\theta$ }]];
  {nC, nq} = Which[
    p >  $\theta$ , {C, q} /. ( $\gamma_i \rightarrow -C[[p]] / \phi[[p]]$ )^+ /. ( $\gamma_i \rightarrow \theta$ )^+,
     $\lambda != \theta$ , (n $\sigma$  += sign[ $\lambda$ ];
    {C, q} /. ( $\gamma_i \rightarrow -(\partial_{\bar{\gamma}_i} q) / \lambda$ )^+ /. ( $\gamma_i \rightarrow \theta$ )^+},
     $\lambda === \theta$ , {C  $\cup$  { $\partial_{\bar{\gamma}_i} q$ }, q} /. ( $\gamma_i \rightarrow \theta$ )^+];
  CF@ $\Sigma_B[\text{Most}[\{ri, li, bs\}] [n\sigma,$ 
    PQ[nC, nq] /. ( $\gamma_{\text{Last}[\{ri, li\}] \rightarrow \gamma_{\text{First}[\{ri, li\}]}$ )^+ ] ]
```

Video: <http://www.math.toronto.edu/~drorbn/Talks/Geneva-231201>. Handout:

<http://www.math.toronto.edu/~drorbn/Talks/USC-240205>.