Strand Operations. c for contract, mc for magnetic contract:

```
c_{i_,j_}@t : Σ_B[{li___,i_,ri___},{___,j_,___},___][__] :=
  t // GT_{j,First@{ri,li}} // Cordon_j
```

```
c_{i_,j_}@t : Σ_B[{___,i_,j_,___},___][__] := Cordon_j@t
c_{i_,j_}@t : Σ_B[{j_,___,i_},___][__] := Cordon_j@t
c_{i_,j_}@t : Σ_B[{___,j_,i_,___},___][__] := Cordon_i@t
c_{i_,j_}@t : Σ_B[{i_,___,j_},___][__] := Cordon_i@t
```

```
mc[ℰ_] := ℰ //.
  t : Σ_B[{___,i_,___},{___,j_,___},___][__] |
      Σ_B[{___,i_,j_,___},___][__] | Σ_B[{j_,___,i_},___][__] /;
    i + j == 0 :> c_{i,j}@t
```

The Crossings (and empty strands).

```
Kas@P_{i_,j_} := CF@Σ_B[{i,j}][0, PQ[{}, 0]];
TL@P_{i_,j_} := CF@Σ_B[{i,j}][0, PQ[{}, 0]]
```

```
Kas[x : X[i_, j_, k_, l_]] :=
  Kas@If[PositiveQ[x], X_{-i,j,k,-l}, X̄_{-j,k,l,-i}];
Kas[(x : X | X̄)_{fs__}] := Module[{v = 2 u^2 - 1, p, γs, m},
  γs = γ_# & /@ {fs}; p = (x === X);
  m = If[p, ( v u 1 u
              u 1 u 1
              1 u v u
              u 1 u 1 ), -( v u 1 u
                            u 1 u 1
                            1 u v u
                            u 1 u 1 )];
  CF@Σ_B[{fs}][If[p, -1, 1], PQ[{}, γs*.m.γs]]]
```

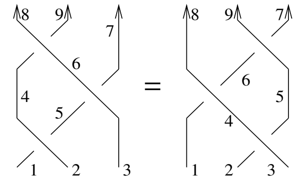
```
TL[x : X[i_, j_, k_, l_]] :=
  TL@If[PositiveQ[x], X_{-i,j,k,-l}, X̄_{-j,k,l,-i}];
TL[(x : X | X̄)_{fs__}] := Module[{t = 1 - ω, r, γs, m},
  r = t + t*; γs = γ_# & /@ {fs};
  m = If[x === X,
    ( -r   -t   2t   t*
      -t*   0   t*   0
      2t*   t   -r   -t*
       t    0   -t   0 ), ( r   -t   -2t*   t*
                            -t*   0    t*    0
                            -2t   t    r    -t*
                             t    0   -t     0 )];
  CF@Σ_B[{fs}][0, PQ[{}, γs*.m.γs]]]
```

Evaluation on Tangles and Knots.

```
Kas[K_] := Fold[mc[#1⊕#2] &, Σ_B[][0, PQ[{}, 0]],
    List @@ (Kas /@ PD@K)];
KasSig[K_] := Expand[Kas[K]⟦1⟧ / 2]
```

```
TL[K_] :=
  Fold[mc[#1⊕#2] &, Σ_B[][0, PQ[{}, 0]],
    List @@ (TL /@ PD@K)] /.
   θ[c_ + u] /; Abs[c] ≥ 1 :> θ[c];
TLSig[K_] := TL[K]⟦1⟧
```

Reidemeister 3.




```
R3L = PD[X_{-2,5,4,-1}, X_{-3,7,6,-5},
    X_{-6,9,8,-4}];
R3R = PD[X_{-3,5,4,-2}, X_{-4,6,8,-1},
    X_{-5,7,9,-6}];
{TL@R3L == TL@R3R, Kas@R3L == Kas@R3R}
```

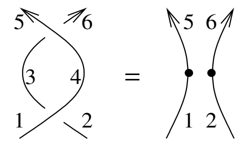
{True, True}

```
Kas@R3L
```

$$2\,\theta\left(u-\tfrac{1}{2}\right)-2\,\theta\left(u+\tfrac{1}{2}\right)-2$$

	$(\gamma_{-3}$	γ_7	γ_9	γ_8	γ_{-1}	$\gamma_{-2})$
$\bar\gamma_{-3}$	$\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$
$\bar\gamma_7$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2(2u^2-1)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$
$\bar\gamma_9$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$
$\bar\gamma_8$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$
$\bar\gamma_{-1}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2(2u^2-1)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$
$\bar\gamma_{-2}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$

Reidemeister 2.




```
TL@PD[X_{-2,4,3,-1}, X̄_{-4,6,5,-3}]
```

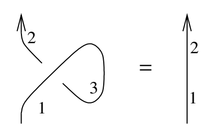
0

	1	0	-1	0
	$(\gamma_{-2}$	γ_6	γ_5	$\gamma_{-1})$
$\bar\gamma_{-2}$	0	0	0	0
$\bar\gamma_6$	0	0	0	0
$\bar\gamma_5$	0	0	0	0
$\bar\gamma_{-1}$	0	0	0	0

```
{TL@PD[X_{-2,4,3,-1}, X̄_{-4,6,5,-3}] == GT_{5,-2}@TL@PD[P_{-1,5}, P_{-2,6}],
 Kas@PD[X_{-2,4,3,-1}, X̄_{-4,6,5,-3}] == GT_{5,-2}@Kas@PD[P_{-1,5}, P_{-2,6}]}
```

{True, True}

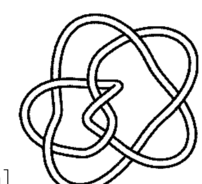
Reidemeister 1.




```
{TL@PD[X_{-3,3,2,-1}] == TL@P_{-1,2},
 Kas@PD[X_{-3,3,2,-1}] == Kas@P_{-1,2}}
```

{True, True}

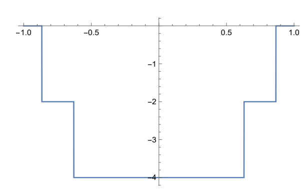
A Knot.



```
f = TLSig[Knot[8, 5]]
```

$$2\,\theta\left[-\frac{\sqrt{3}}{2}+u\right]-2\,\theta\left[\frac{\sqrt{3}}{2}+u\right]-2\,\theta\left[u-\boxed{-0.630\ldots}\right]+2\,\theta\left[u-\boxed{0.630\ldots}\right]$$

```
Plot[f, {u, -1, 1}]
```



Video: http://www.math.toronto.edu/~drorbn/Talks/Geneva-231201. Handout: http://www.math.toronto.edu/~drorbn/Talks/USC-240205.