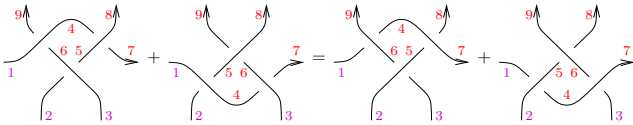


Conway's Third Identity

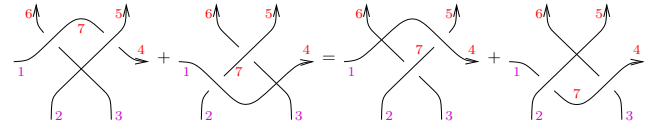
(see [Co])



$$\mathcal{A}@\{X_{6,4,9,1}, \bar{X}_{4,5,7,8}, \bar{X}_{2,3,5,6}\} + \mathcal{A}@\{X_{2,4,5,1}, \bar{X}_{4,3,7,6}, X_{6,8,9,5}\} \equiv \mathcal{A}@\{\bar{X}_{1,6,4,9}, X_{5,7,8,4}, X_{3,5,6,2}\} + \mathcal{A}@\{\bar{X}_{1,2,4,5}, X_{3,7,6,4}, \bar{X}_{5,6,8,9}\}$$

True

Virtual version (Archibald, [Ar])

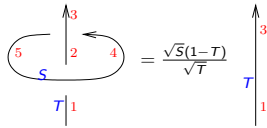


$$\mathcal{A}@\{X_{3,7,6,1}, \bar{X}_{7,2,4,5}\} + \mathcal{A}@\{X_{2,4,7,1}, X_{3,5,6,7}\} \equiv \mathcal{A}@\{X_{3,7,6,2}, X_{7,4,5,1}\} + \mathcal{A}@\{\bar{X}_{1,2,7,5}, X_{3,4,6,7}\}$$

True

Jun Murakami's Fifth Axiom

(see [Mu])

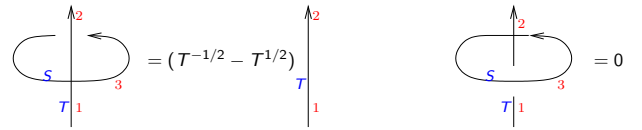


$$\mathcal{A}@\{X_{1,4,2,5}[T, S], X_{4,3,5,2}\} \equiv \frac{\sqrt{S}(1-T)}{\sqrt{T}} \mathcal{A}@\{P_{1,3}[T]\}$$

True



Virtual versions (Archibald, [Ar])



$$\mathcal{A}@\{X_{3,2,3,1}[S, T]\} \equiv (T^{-1/2} - T^{1/2}) \mathcal{A}@\{P_{1,2}[T]\}$$

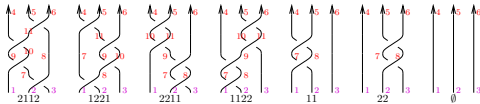
True

$$\mathcal{A}@\{X_{1,3,2,3}\}$$

$$\mathcal{A}[\{1\}, \{2\}, \langle \xi_1 \rightarrow \tau_1, \xi_2 \rightarrow \tau_1 \rangle, \emptyset]$$

Jun Murakami's Third Axiom

(see [Mu])

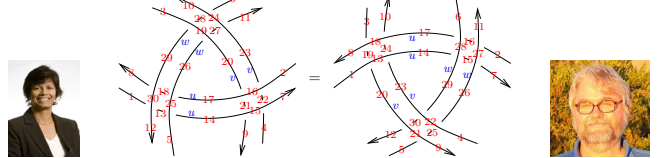


$$\begin{aligned} \mathcal{A}_{2112} &= \mathcal{A}@\{X_{3,8,7,2}, X_{7,10,9,1}, X_{10,11,4,9}, X_{8,6,5,11}\}; \\ \mathcal{A}_{1221} &= \mathcal{A}@\{X_{2,8,7,1}, X_{3,10,9,8}, X_{10,6,11,9}, X_{11,5,4,7}\}; \\ \mathcal{A}_{2211} &= \mathcal{A}@\{X_{3,8,7,2}, X_{8,6,9,7}, X_{9,11,10,1}, X_{11,5,4,10}\}; \\ \mathcal{A}_{1122} &= \mathcal{A}@\{X_{2,8,7,1}, X_{8,9,4,7}, X_{3,11,10,9}, X_{11,6,5,10}\}; \\ \mathcal{A}_{11} &= \mathcal{A}@\{X_{2,8,7,1}, X_{8,5,4,7}, P_{3,6}\}; \quad \mathcal{A}_{22} = \mathcal{A}@\{X_{3,8,7,2}, X_{8,6,5,7}, P_{1,4}\}; \\ \mathcal{A}_\emptyset &= \mathcal{A}@\{P_{1,4}, P_{2,5}, P_{3,6}\}; \\ g_+ [z_-] &:= z^{1/2} + z^{-1/2}; \quad g_- [z_-] := z^{1/2} - z^{-1/2}; \\ g_- [\tau_1] g_- [\tau_2] \mathcal{A}_{2112} - g_- [\tau_2] g_- [\tau_3] \mathcal{A}_{1221} - g_- [\tau_3 / \tau_1] (\mathcal{A}_{2211} + \mathcal{A}_{1122}) + \\ &g_- [\tau_2 \tau_3 / \tau_1] g_+ [\tau_3] \mathcal{A}_{11} - g_- [\tau_1] g_- [\tau_2 / \tau_3] \mathcal{A}_{22} \equiv g_- [\tau_3^2 / \tau_1^2] \mathcal{A}_\emptyset \end{aligned}$$

True

The Naik-Stanford Double Delta Move

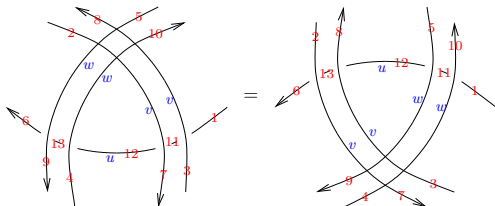
(see [NS])



$$\begin{aligned} \text{Timing}[\mathcal{A}@\{X_{6,10,28,24}[w, v], \bar{X}_{28,3,29,19}[w, v], X_{26,20,27,19}[w, v], \bar{X}_{27,23,11,24}[w, v], \\ X_{1,12,13,30}[u, w], \bar{X}_{13,5,14,25}[u, w], X_{17,26,18,25}[u, w], \bar{X}_{18,29,8,30}[u, w], \\ X_{4,7,22,15}[v, u], \bar{X}_{22,2,23,16}[v, u], X_{20,17,21,16}[v, u], \bar{X}_{21,14,9,15}[v, u]\}] \equiv \\ \mathcal{A}@\{X_{5,9,25,21}[w, v], \bar{X}_{25,4,26,22}[w, v], X_{29,23,30,22}[w, v], \bar{X}_{30,20,12,21}[w, v], \\ X_{2,11,16,27}[u, w], \bar{X}_{16,6,17,28}[u, w], X_{14,29,15,28}[u, w], \bar{X}_{15,26,7,27}[u, w], \\ X_{3,8,19,18}[v, u], \bar{X}_{19,1,20,13}[v, u], X_{23,14,24,13}[v, u], \bar{X}_{24,17,10,18}[v, u]\}] \end{aligned}$$

{190.422, True}

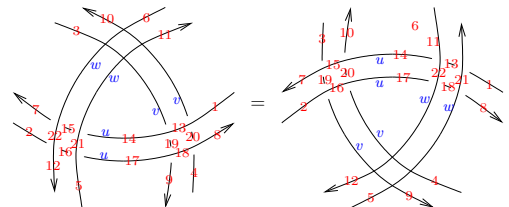
Virtual Version 1 (Archibald, [Ar])



$$\mathcal{A}@\{X_{1,8,11,3}[u, v], \bar{X}_{11,2,12,7}[u, v], X_{12,10,13,4}[u, w], \bar{X}_{13,5,6,9}[u, w]\} \equiv \mathcal{A}@\{X_{1,10,11,4}[u, w], \bar{X}_{11,5,12,9}[u, w], X_{12,8,13,3}[u, v], \bar{X}_{13,2,6,7}[u, v]\}$$

True

Virtual Version 2 (Archibald, [Ar])



$$\begin{aligned} \mathcal{A}@\{\bar{X}_{20,1,10,13}[v, u], X_{3,14,19,13}[v, u], X_{14,11,15,21}[u, w], \bar{X}_{15,6,7,22}[u, w], \\ X_{2,12,16,22}[u, w], \bar{X}_{16,5,17,21}[u, w], \bar{X}_{19,17,9,18}[v, u], X_{4,8,20,18}[v, u]\} \equiv \\ \mathcal{A}@\{X_{1,11,13,21}[u, w], \bar{X}_{13,6,14,22}[u, w], \bar{X}_{20,14,10,15}[v, u], X_{3,7,19,15}[v, u], \\ \bar{X}_{19,2,9,16}[v, u], X_{4,17,20,16}[v, u], X_{17,12,18,22}[u, w], \bar{X}_{18,5,8,21}[u, w]\} \end{aligned}$$

True