Define[lhs = rhs, ...] defines the lhs to be rhs, except that rhs is computed only once for each value of $k. Fancy Mathematica not for the faint of heart. Most readers should ignore.

```
SetAttributes[Define, HoldAll];
Define[def_, defs__] := (Define[def]; Define[defs];);
Define[op_is__ = ℰ_] :=
 Module[{SD, ii, jj, kk, isp, nis, nisp, sis},
  Block[{i, j, k},
   ReleaseHold[Hold[
      SD[op_nisp,$k_Integer, PP_Boot@Block[{i, j, k}, op_isp,$k = ℰ;
         op_nis,$k]];
      SD[op_isp, op_{is},$k]; SD[op_sis__, op_{sis}];
     ] /. {SD → SetDelayed,
      isp → {is} /. {i → i_, j → j_, k → k_},
      nis → {is} /. {i → ii, j → jj, k → kk},
      nisp → {is} /. {i → ii_, j → jj_, k → kk_}
     }] ]]
```

The Objects

ωεβ/objects

Symmetric Algebra Objects

```
sm_i_,j_→k_ :=
  𝔼{i,j}→{k}[b_k (β_i + β_j) + t_k (τ_i + τ_j) + a_k (α_i + α_j) +
    y_k (η_i + η_j) + x_k (ξ_i + ξ_j)];
sΔ_i_→j_,k_ :=
  𝔼{i}→{j,k}[β_i (b_j + b_k) + τ_i (t_j + t_k) + α_i (a_j + a_k) +
    η_i (y_j + y_k) + ξ_i (x_j + x_k)];
sS_i_ := 𝔼{i}→{i}[-β_i b_i - τ_i t_i - α_i a_i - η_i y_i - ξ_i x_i];
sϵ_i_ := 𝔼{}→{i}[0];
sη_i_ := 𝔼{i}→{}[0];
sσ_i_→j_ := 𝔼{i}→{j}[β_i b_j + τ_i t_j + α_i a_j + η_i y_j + ξ_i x_j];
sϒ_i_→j_,k_,l_,m_ := 𝔼{i}→{j,k,l,m}[β_i b_k + τ_i t_k + α_i a_l + η_i y_j + ξ_i x_m];
```

The CU Definitions

$$c\Lambda = \left(\eta_i + \frac{e^{-\gamma\alpha_i-\epsilon\beta_i}\eta_j}{1+\gamma\epsilon\eta_j\xi_i}\right) y_k + \left(\beta_i+\beta_j+\frac{\text{Log}[1+\gamma\epsilon\eta_j\xi_i]}{\epsilon}\right) b_k + \left(\alpha_i+\alpha_j+\frac{\text{Log}[1+\gamma\epsilon\eta_j\xi_i]}{\gamma}\right) a_k + \left(\frac{e^{-\gamma\alpha_j-\epsilon\beta_j}\xi_i}{1+\gamma\epsilon\eta_j\xi_i}+\xi_j\right) x_k;$$

```
Define[cm_i,j→k = 𝔼{i,j}→{k}[cΛ]]
Define[cσ_i→j = sσ_i,j /. τ_i → 0, cϵ_i = sϵ_i, cη_i = sη_i,
  cΔ_i→j,k = sΔ_i→j,k,
  cS_i = sS_i // sϒ_i→1,2,3,4 // cm_4,3→i // cm_i,2→i // cm_i,1→i];
```

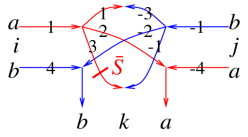
Booting Up QU

```
Define[aσ_i→j = 𝔼{i}→{j}[a_j α_i + x_j ξ_i],
 bσ_i→j = 𝔼{i}→{j}[b_j β_i + y_j η_i]]
Define[am_i,j→k = 𝔼{i,j}→{k}[(α_i + α_j) a_k + (𝒜_j^-1 ξ_i + ξ_j) x_k],
 bm_i,j→k = 𝔼{i,j}→{k}[(β_i + β_j) b_k + (η_i + e^-ϵ β_i η_j) y_k]]
Define[R_i,j = 𝔼{}→{i,j}[ħ a_j b_i + Sum[k,1,$k+1] (1 - e^γ∈ħ)^k (ħ y_i x_j)^k / (k (1 - e^k γ∈ħ))],
 R̄_i,j = CF@𝔼{}→{i,j}[-ħ a_j b_i, -ħ x_j y_i / B_i,
    1 + If[$k == 0, 0, (R̄_{i,j},$k-1)$k[3] -
       (((R̄_{i,j},0)$k R_1,2 (R̄_{3,4},$k-1)$k) // (bm_i,1→i am_j,2→j) //
         (bm_i,3→i am_j,4→j))[3]]],
 P_i,j = 𝔼{i,j}→{}[β_i α_j / ħ, η_i ξ_j / ħ,
   1 + If[$k == 0, 0, (P_{i,j},$k-1)$k[3] -
      (R_1,2 // ((P_{1,j},0)$k (P_{i,2},$k-1)$k))[3]]]
```

```
Define[aS_i = (aσ_i→2 R̄_1,i) // P_1,2,
 āS_i = 𝔼{i}→{i}[-a_i α_i, -x_i 𝒜_i ξ_i,
   1 + If[$k == 0, 0, (āS_{i},$k-1)$k[3] -
      ((āS_{i},0)$k // aS_i // (āS_{i},$k-1)$k)[3]]]]
```

```
Define[bS_i = bσ_i→1 R_i,2 // aS_2 // P_1,2,
 b̄S_i = bσ_i→1 R_i,2 // āS_2 // P_1,2,
 aΔ_i→j,k = (R_1,j R_2,k) // bm_1,2→3 // P_3,i,
 bΔ_i→j,k = (R_j,1 R_k,2) // am_1,2→3 // P_i,3]
```

The Drinfel'd double:




```
Define[
 dm_i,j→k =
  ((sϒ_i→4,4,1,1 // aΔ_1→1,2 // aΔ_2→2,3 // āS_3)
     (sϒ_j→-1,-1,-4,-4 // bΔ_-1→-1,-2 // bΔ_-2→-2,-3)) //
   (P_-1,3 P_-3,1 am_2,-4→k bm_4,-2→k)]
```

```
Define[dσ_i→j = aσ_i→j bσ_i→j,
 dϵ_i = sϵ_i, dη_i = sη_i,
 dS_i = sϒ_i→1,1,2,2 // (b̄S_1 aS_2) // dm_2,1→i,
 d̄S_i = sϒ_i→1,1,2,2 // (bS_1 āS_2) // dm_2,1→i,
 dΔ_i→j,k = (bΔ_i→3,1 aΔ_i→2,4) // (dm_3,4→k dm_1,2→j)]
```

```
Define[C_i = 𝔼{}→{i}[0, 0, B_i^1/2 e^-ħ∈a_i/2]_$k,
 C̄_i = 𝔼{}→{i}[0, 0, B_i^-1/2 e^ħ∈a_i/2]_$k,
 Kink_i = (R_1,3 C̄_2) // dm_1,2→1 // dm_1,3→i,
 K̄ink_i = (R̄_1,3 C_2) // dm_1,2→1 // dm_1,3→i]
```

Note. $t == \epsilon a - \gamma b$ and $b == -t/\gamma + \epsilon a/\gamma$.

```
Define[b2t_i = 𝔼{i}→{i}[α_i a_i + β_i (ϵ a_i - t_i) / γ + ξ_i x_i + η_i y_i],
 t2b_i = 𝔼{i}→{i}[α_i a_i + τ_i (ϵ a_i - γ b_i) + ξ_i x_i + η_i y_i]]
```

The Knot Tensors

```
Define[kR_i,j = R_i,j // (b2t_i b2t_j) /. t_i|j → t,
 k̄R_i,j = R̄_i,j // (b2t_i b2t_j) /. {t_i|j → t, T_i|j → T},
 km_i,j→k = (t2b_i t2b_j) // dm_i,j→k //
   b2t_k /. {t_k → t, T_k → T, τ_i|j → 0},
 kC_i = C_i // b2t_i /. T_i → T,
 k̄C_i = C̄_i // b2t_i /. T_i → T,
 kKink_i = Kink_i // b2t_i /. {t_i → t, T_i → T},
 k̄Kink_i = K̄ink_i // b2t_i /. {t_i → t, T_i → T}]
```

Some of the Atoms.

ωεβ/atoms

With $\mathcal{A}_i := e^{\alpha_i}$ and $B_i = e^{-b_i}$,

```
PP_ := Identity; $k = 1; ħ = γ = 1;
Column[
 (# → (ℰ = ToExpression[#];
       Normal@Simplify[ℰ[[1]] + ℰ[[2]] + Log@ℰ[[3]]])) & /@
  {"dm_i,j→k", "dΔ_i→j,k", "dS_i", "R_i,j", "P_i,j"}]
```

Video and more: http://www.math.toronto.edu/~drorbn/Talks/CRM-1907, http://www.math.toronto.edu/~drorbn/Talks/UCLA-191101.