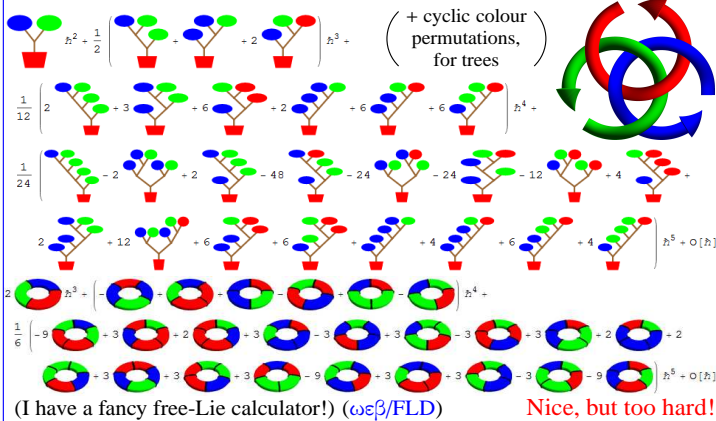


Theorem 2 [BND]. $\exists!$ a homomorphic expansion, aka a homomorphic universal finite type invariant Z^w of pure w-tangles. $z^w := \log Z^w$ takes values in $FL(S)^S \times CW(S)$.

z is computable. z of the Borromean tangle, to degree 5 [BN]:



Definition. (Compare [BNS, BN]) A **The Abstract Context**

meta-monoid is a functor $M: (\text{finite sets, injections}) \rightarrow (\text{sets})$ (think “ $M(S)$ is quantum G^S ”, for G a group) along with natural operations $*$: $M(S_1) \times M(S_2) \rightarrow M(S_1 \sqcup S_2)$ whenever $S_1 \cap S_2 = \emptyset$ and $m_c^{ab}: M(S) \rightarrow M((S \setminus \{a, b\}) \sqcup \{c\})$ whenever $a \neq b \in S$ and $c \notin S \setminus \{a, b\}$, such that

$$\text{meta-associativity: } m_a^{ab} // m_a^{ac} = m_b^{bc} // m_a^{ab}$$

$$\text{meta-locality: } m_c^{ab} // m_f^{de} = m_f^{de} // m_c^{ab}$$

and, with $\epsilon_b = M(S \hookrightarrow S \sqcup \{b\})$,

$$\text{meta-unit: } \epsilon_b // m_a^{ab} = Id = \epsilon_b // m_a^{ba}$$

Claim. Pure virtual tangles PT form a meta-monoid.

Theorem. $S \mapsto \Gamma_0(S)$ is a meta-monoid and $z_0: PT \rightarrow \Gamma_0$ is a morphism of meta-monoids.

Strong Conviction. There exists an extension of Γ_0 to a bigger meta-monoid $\Gamma_{01}(S) = \Gamma_0(S) \times \Gamma_1(S)$, along with an extension of z_0 to $z_{01}: PT \rightarrow \Gamma_{01}$, with

$$\Gamma_1(S) = V \oplus V^{\otimes 2} \oplus V^{\otimes 3} \oplus S^2(V)^{\otimes 2} \quad (\text{with } V := R_S(S)).$$

Furthermore, upon reducing to a single variable everything is polynomial size and polynomial time.

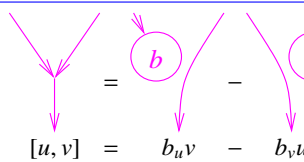
Furthermore, Γ_{01} is given using a “meta-2-cocycle ρ_c^{ab} over Γ_0 ”: In addition to $m_c^{ab} \rightarrow m_{0c}^{ab}$, there are R_S -linear $m_{1c}^{ab}: \Gamma_1(S \sqcup \{a, b\}) \rightarrow \Gamma_1(S \sqcup \{c\})$, a meta-right-action $\alpha^{ab}: \Gamma_1(S) \times \Gamma_0(S) \rightarrow \Gamma_1(S)$ R_S -linear in the first variable, and a first order differential operator (over R_S) $\rho_c^{ab}: \Gamma_0(S \sqcup \{a, b\}) \rightarrow \Gamma_1(S \sqcup \{c\})$ such that

$$(\zeta_0, \zeta_1) // m_c^{ab} = (\zeta_0 // m_{0c}^{ab}, (\zeta_1, \zeta_0) // \alpha^{ab} // m_{1c}^{ab} + \zeta_0 // \rho_c^{ab})$$

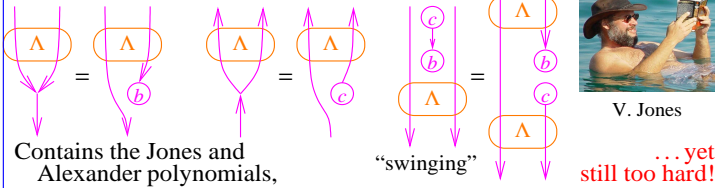
What's done? The braid part, with still-ugly formulas.

What's missing? A lot of concept- and detail-sensitive work towards m_{1c}^{ab} , α^{ab} , and ρ_c^{ab} . The “ribbon element”.

Proposition [BN]. Modulo all relations that universally hold for the 2D non-Abelian Lie algebra and after some changes-of-variable, z^w reduces to z_0 .



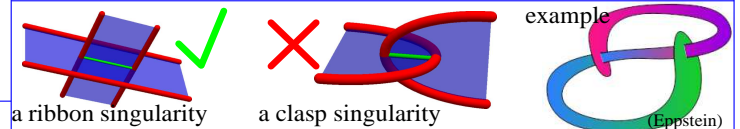
Back to v – the 2D “Jones Quotient”.



The OneCo Quotient. Likely related to [ADO] $= 0$, only one co-bracket is allowed. Everything should work, and everything is being worked!

References.

[ADO] Y. Akutsu, T. Deguchi, and T. Ohtsuki, *Invariants of Colored Links*, J. of Knot Theory and its Ramifications **1-2** (1992) 161–184.
 [BN] D. Bar-Natan, *Balloons and Hoops and their Universal Finite Type Invariant, BF Theory, and an Ultimate Alexander Invariant*, $\omega\epsilon\beta$ /KBH, arXiv:1308.1721.
 [BND] D. Bar-Natan and Z. Dancso, *Finite Type Invariants of W-Knotted Objects I-II*, $\omega\epsilon\beta$ /WKO1, $\omega\epsilon\beta$ /WKO2, arXiv:1405.1956, arXiv:1405.1955.
 [BNS] D. Bar-Natan and S. Selmani, *Meta-Monoids, Meta-Bicrossed Products, and the Alexander Polynomial*, J. of Knot Theory and its Ramifications **22-10** (2013), arXiv:1302.5689.
 [CT] D. Cimasoni and V. Turaev, *A Lagrangian Representation of Tangles*, Topology **44** (2005) 747–767, arXiv:math.GT/0406269.
 [En] B. Enriquez, *A Cohomological Construction of Quantization Functors of Lie Bialgebras*, Adv. in Math. **197-2** (2005) 430–479, arXiv:math/0212325.
 [EK] P. Etingof and D. Kazhdan, *Quantization of Lie Bialgebras, I*, Selecta Mathematica **2** (1996) 1–41, arXiv:q-alg/9506005.
 [GST] R. E. Gompf, M. Scharlemann, and A. Thompson, *Fibered Knots and Potential Counterexamples to the Property 2R and Slice-Ribbon Conjectures*, Geom. and Top. **14** (2010) 2305–2347, arXiv:1103.1601.
 [KLW] P. Kirk, C. Livingston, and Z. Wang, *The Gassner Representation for String Links*, Comm. Cont. Math. **3** (2001) 87–136, arXiv:math/9806035.
 [LD] J. Y. Le Dimet, *Enlacements d'Intervalles et Représentation de Gassner*, Comment. Math. Helv. **67** (1992) 306–315.

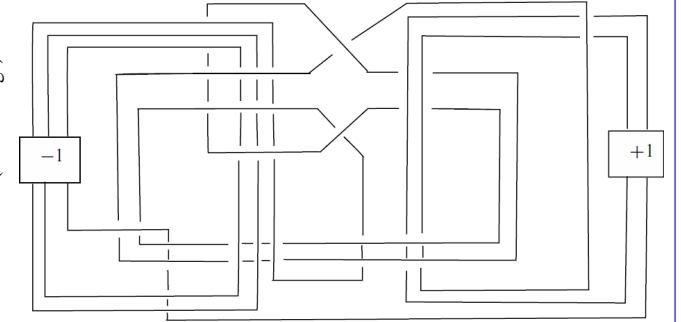


A bit about ribbon knots. A “ribbon knot” is a knot that can be presented as the boundary of a disk that has “ribbon singularities”, but no “clasp singularities”. A “slice knot” is a knot in $S^3 = \partial B^4$ which is the boundary of a non-singular disk in B^4 . Every ribbon knots is clearly slice, yet,

Conjecture. Some slice knots are not ribbon.

Fox-Milnor. The Alexander polynomial of a ribbon knot is always of the form $A(t) = f(t)f(1/t)$. (also for slice)

[GST]: a slice knot that might not be ribbon (48 crossings).



Help Needed!
 I'm slow and feeble-minded.



“God created the knots, all else in topology is the work of mortals.”
 Leopold Kronecker (modified)

