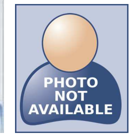
A Partial Reduction of BF Theory to Combinatorics, 1

Abstract. I will describe a semi-rigorous reduction of perturbative BF theory (Cattaneo-Rossi [CR]) to computable combinatorics, in the case of ribbon 2-links. Also, I will explain how and why my approach may or may not work in the non-ribbon case. Weak this result is, and at least partially already known (Watanabe [Wa]). Yet in the ribbon case, the resulting invariant is a universal finite type invariant, a gadget that significantly generalizes and clarifies the Alexander polynomial and that is closely related to the Kashiwara-Vergne problem. I cannot rule out the possibility that the corresponding gadget in the non-ribbon case will be as interesting. (good news in highlight)

The BF Feynman Rules. For an edge e, let Φ_e be its direction, in S^3 or S^1. Let ω_3 and ω_1 be volume forms on S^3 and S_1. Then for a 2-link $(\kappa_t)_{t\in T}$,


Cattaneo



Rossi

$$\zeta = \log \sum_{\substack{\text{diagrams}\\ D}} \frac{[D]}{|\mathrm{Aut}(D)|} \underbrace{\int_{\mathbb{R}^2}\cdots\int_{\mathbb{R}^2}}_{S\text{-vertices}} \underbrace{\int_{\mathbb{R}^4}\cdots\int_{\mathbb{R}^4}}_{M\text{-vertices}} \prod_{\substack{\text{red}\\ e\in D}} \Phi_e^*\omega_3 \prod_{\substack{\text{black}\\ e\in D}} \Phi_e^*\omega_1$$

is an invariant in $CW(FL(T)) \to CW(T)/\sim$, "symmetrized cyclic words in T".

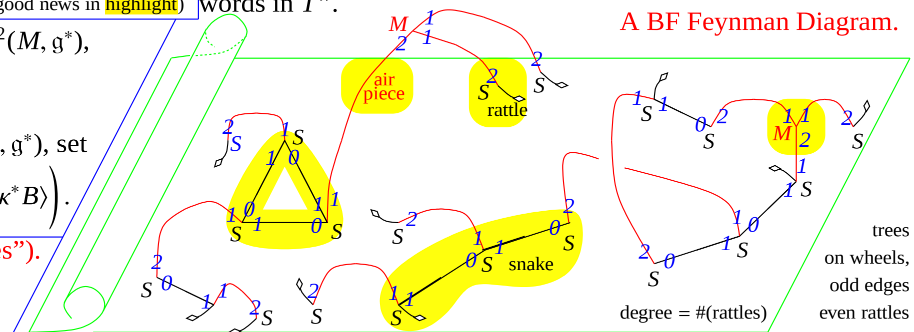
BF Following [CR]. $A \in \Omega^1(M = \mathbb{R}^4, \mathfrak{g})$, $B \in \Omega^2(M, \mathfrak{g}^*)$,

$$S(A,B) := \int_M \langle B, F_A\rangle.$$

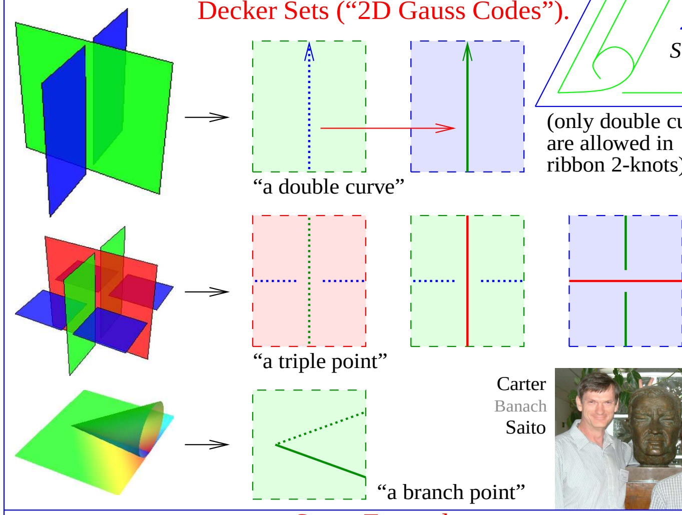
With $\kappa\colon (S = \mathbb{R}^2) \to M$, $\beta \in \Omega^0(S, \mathfrak{g})$, $\alpha \in \Omega^1(S, \mathfrak{g}^*)$, set

$$O(A,B,\kappa) := \int \mathcal{D}\beta\mathcal{D}\alpha \exp\left(\frac{i}{\hbar}\int_S \langle\beta, d_{\kappa^*A}\alpha + \kappa^*B\rangle\right).$$

A BF Feynman Diagram.



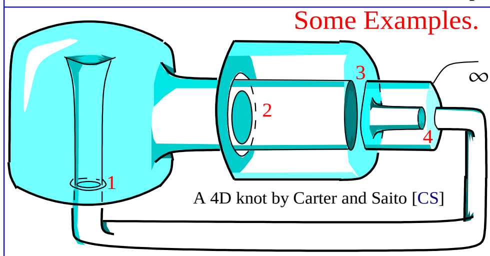
Decker Sets ("2D Gauss Codes").



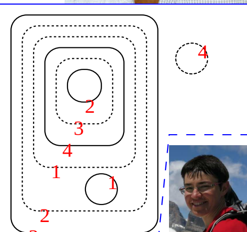
(only double curves are allowed in ribbon 2-knots)

Carter
Banach
Saito

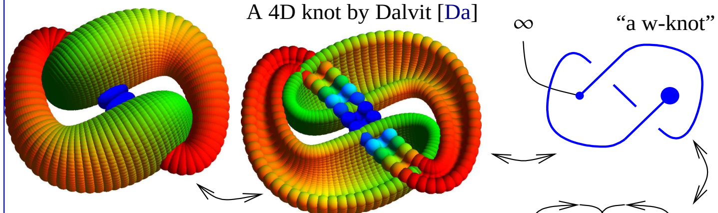
Some Examples.

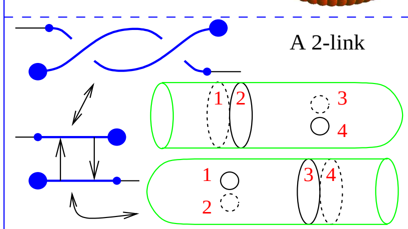


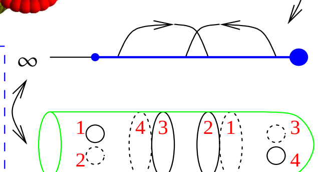
A 4D knot by Carter and Saito [CS]


Dalvit

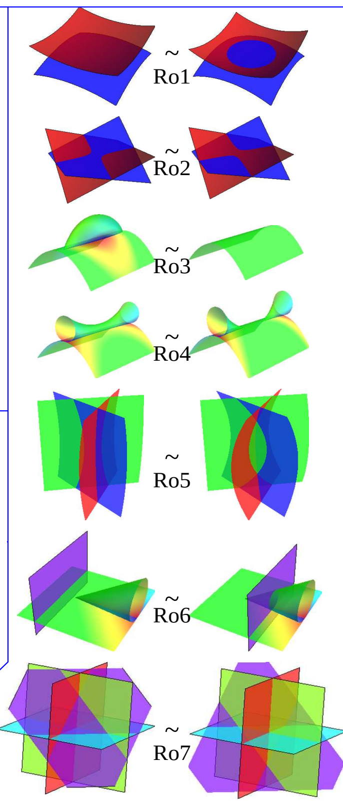
A 4D knot by Dalvit [Da]

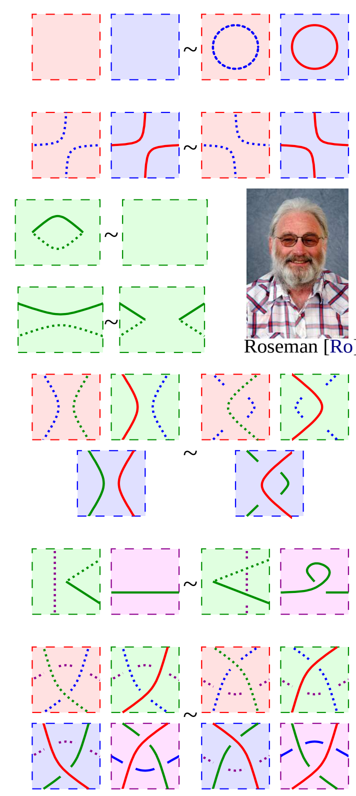



A 2-link

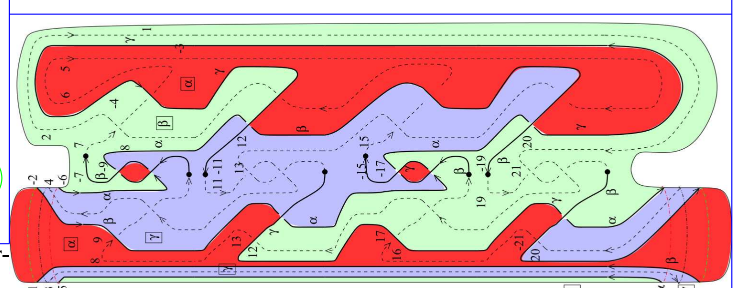


"ribbed cigar presentation"





Roseman [Ro]



A 2-twist spun trefoil by Carter-Kamada-Saito [CKS].