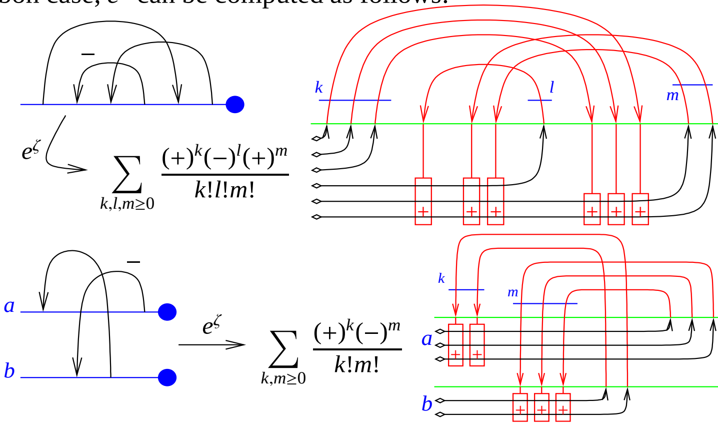
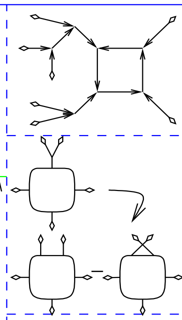
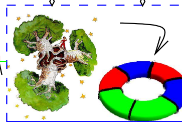
A Partial Reduction of BF Theory to Combinatorics, 2

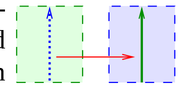
Theorem 1 (with Cattaneo, Dalvit (credit, no blame)). In the ribbon case, e^ζ can be computed as follows:

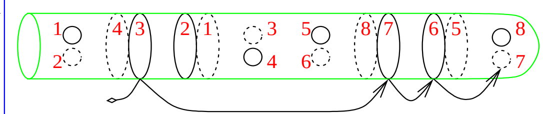


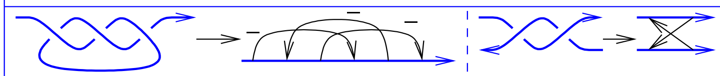




Sketch of Proof. In 4D axial gauge, only "drop down" red propagators, hence in the ribbon case, no M-trivalent vertices. S integrals are ± 1 iff "ground pieces" run on nested curves as below, and exponentials arise when several propagators compete for the same double curve. And then the combinatorics is obvious…





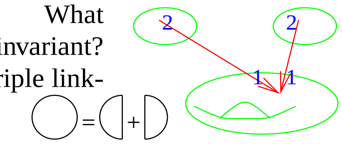


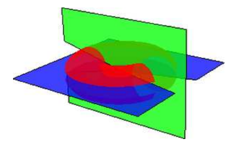
Theorem 2. Using Gauss diagrams to represent knots and T-component pure tangles, the above formulas define an invariant in $CW(FL(T)) \to CW(T)$, "cyclic words in T".
• Agrees with BN-Dancso [BND] and with [BN2]. • In-practice computable! • Vanishes on braids. • Extends to w. • Contains Alexander. • The "missing factor" in Levine's factorization [Le] (the rest of [Le] also fits, hence contains the MVA). • Related to / extends Farber's [Fa]? • Should be summed and categorified.

Musings

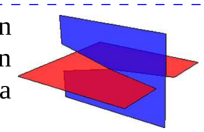
Chern-Simons. When the domain of BF is restricted to ribbon knots, and the target of Chern-Simons is restricted to trees and wheels, they agree. Why?

Is this all? What about the $\vee$-invariant? (the "true" triple linking number)

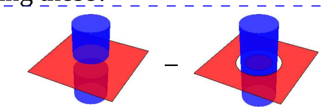




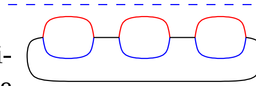
Gnots. In 3D, a generic immersion of S^1 is an embedding, a knot. In 4D, a generic immersion of a surface has finitely-many double points (a gnot?). Perhaps we should be studying these?



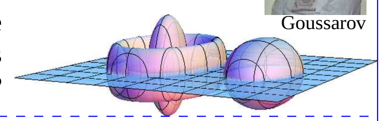
Finite type. What are finite-type invariants for 2-knots? What would be "chord diagrams"?



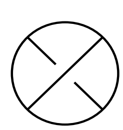
Bubble-wrap-finite-type. There's an alternative definition of finite type in 3D, due to Goussarov (see [BN1]). The obvious parallel in 4D involves "bubble wraps". Is it any good?

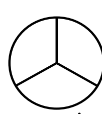


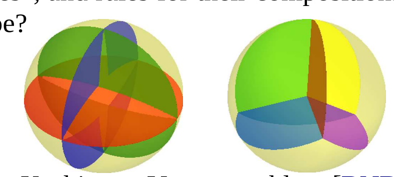

Goussarov



Shielded tangles. In 3D, one can't zoom in and compute "the Chern-Simons invariant of a tangle". Yet there are well-defined invariants of "shielded tangles", and rules for their compositions. What would the 4D analog be?

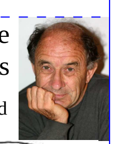



"an associator"



Will the relationship with the Kashiwara-Vergne problem [BND] necessarily arise here?

Plane curves. Shouldn't we understand integral / finite type invariants of plane curves, in the style of Arnold's J^+, J^-, and St [Ar], a bit better?


Arnold

St	1	0	0	0	0	1	2	3	. . .
J^+	0	2	0	0	0	−2	−4	−6	. . .
J^-	0	0	−2	−1	0	−3	−6	−9	. . .

"God created the knots, all else in topology is the work of mortals."
Leopold Kronecker (modified)

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Continuing Joost Slingerland…

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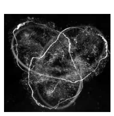

Video and more at http://www.math.toronto.edu/~drorbn/Talks/Vienna-1402/