

The Kauffman Bracket:  $\langle \emptyset \rangle = 1$ ;  $\langle \bigcirc L \rangle = (q + q^{-1})\langle L \rangle$ ;  $\langle \times \rangle = \langle \underset{0\text{-smoothing}}{\times} \rangle - q \langle \underset{1\text{-smoothing}}{\times} \rangle$ .

The Jones Polynomial:  $\hat{J}(L) = (-1)^{n_-} q^{n_+ - 2n_-} \langle L \rangle$ , where  $(n_+, n_-)$  count  $(\times, \times)$  crossings.

Khovanov's construction:  $[[L]]$  — a chain complex of graded  $\mathbb{Z}$ -modules;

$$[[\emptyset]] = 0 \rightarrow \underset{\text{height } 0}{\mathbb{Z}} \rightarrow 0; \quad [[\bigcirc L]] = V \otimes [[L]]; \quad [[\times]] = \text{Flatten} \left( 0 \rightarrow \underset{\text{height } 0}{[[\times]]} \rightarrow \underset{\text{height } 1}{[[\times]]\{1\}} \rightarrow 0 \right);$$

$$\mathcal{H}(L) = \mathcal{H}(\mathcal{C}(L) = [[L]][-n_-]\{n_+ - 2n_-\})$$

$$V = \text{span}\langle v_+, v_- \rangle; \quad \deg v_{\pm} = \pm 1; \quad q\dim V = q + q^{-1} \quad \text{with} \quad q\dim \mathcal{O} := \sum_m q^m \dim \mathcal{O}_m;$$

$$\mathcal{O}\{l\}_m := \mathcal{O}_{m-l} \quad \text{so} \quad q\dim \mathcal{O}\{l\} = q^l q\dim \mathcal{O}; \quad \cdot[s]: \text{height shift by } s;$$

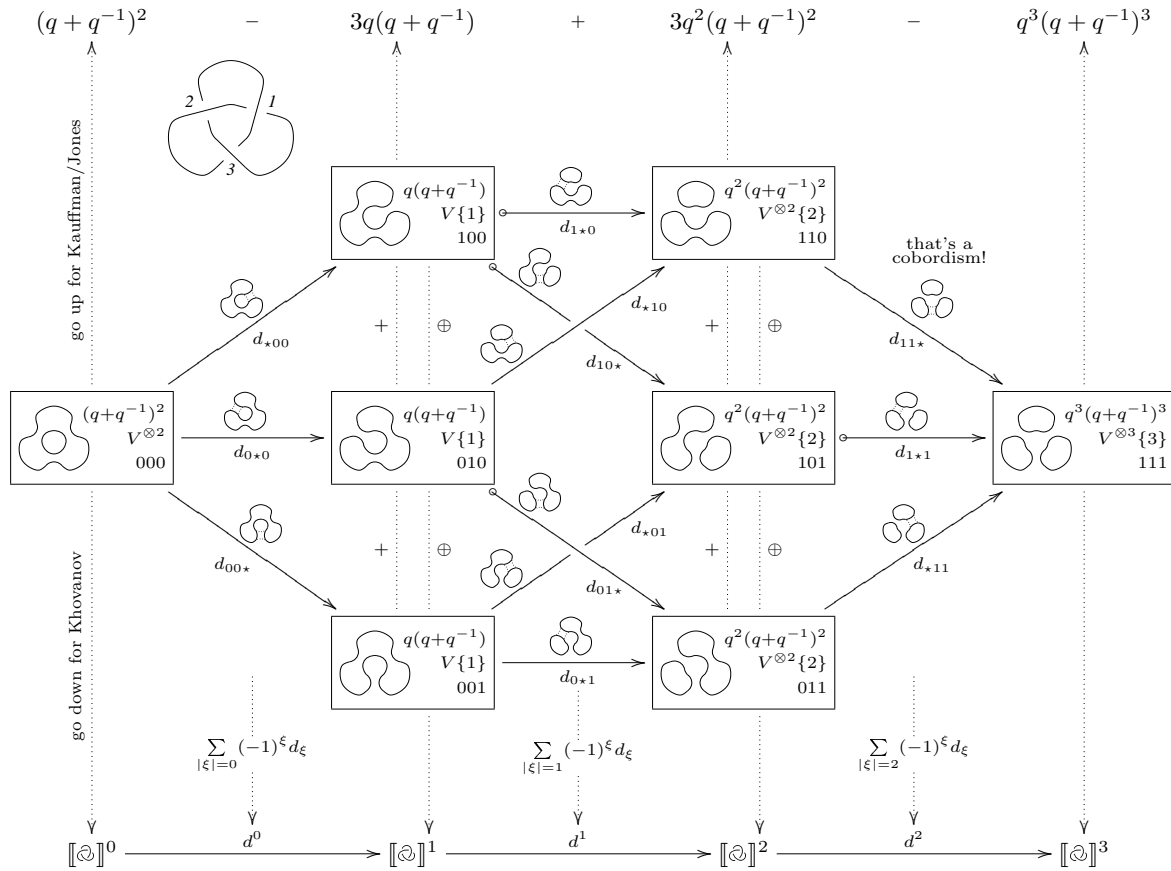
$$\left( \begin{array}{c} \bigcirc \bigcirc \xrightarrow{\quad} \bigcirc \bigcirc \\ \bigcirc \bigcirc \xrightarrow{\quad} \bigcirc \bigcirc \end{array} \right) \rightarrow (V \otimes V \xrightarrow{m} V) \quad m: \begin{cases} v_+ \otimes v_- \mapsto v_- & v_+ \otimes v_+ \mapsto v_+ \\ v_- \otimes v_+ \mapsto v_- & v_- \otimes v_- \mapsto 0 \end{cases}$$

$$\left( \begin{array}{c} \bigcirc \bigcirc \xrightarrow{\quad} \bigcirc \bigcirc \\ \bigcirc \bigcirc \xrightarrow{\quad} \bigcirc \bigcirc \end{array} \right) \rightarrow (V \xrightarrow{\Delta} V \otimes V) \quad \Delta: \begin{cases} v_+ \mapsto v_+ \otimes v_- + v_- \otimes v_+ \\ v_- \mapsto v_- \otimes v_- \end{cases}$$

That's a Frobenius Algebra! And a  $(1+1)$ -dimensional TQFT!

Example:

$$\rho \quad q^{-2} + 1 + q^2 - q^6 \xrightarrow[\text{(with } (n_+, n_-) = (3, 0))]{\cdot (-1)^{n_-} q^{n_+ - 2n_-}} q + q^3 + q^5 - q^9.$$



$$\text{(here } (-1)^\xi := (-1)^{\sum_{i < j} \xi_i} \text{ if } \xi_j = \star) \quad = \quad [[\otimes]] \xrightarrow[\text{(with } (n_+, n_-) = (3, 0))]{\cdot [-n_-]\{n_+ - 2n_-\}} \mathcal{C}(\otimes).$$

**Theorem 1.** The graded Euler characteristic of  $\mathcal{C}(L)$  is  $\hat{J}(L)$ .

**Theorem 2.** The homology  $\mathcal{H}(L)$  is a link invariant and thus so is  $Kh_{\mathbb{F}}(L) := \sum_r t^r q\dim \mathcal{H}_{\mathbb{F}}^r(\mathcal{C}(L))$  over any field  $\mathbb{F}$ .

**Theorem 3.**  $\mathcal{H}(\mathcal{C}(L))$  is strictly stronger than  $\hat{J}(L)$ :  $\mathcal{H}(\mathcal{C}(\bar{5}_1)) \neq \mathcal{H}(\mathcal{C}(10_{132}))$  whereas  $\hat{J}(\bar{5}_1) = \hat{J}(10_{132})$ .

**Conjecture 1.**  $Kh_{\mathbb{Q}}(L) = q^{s-1} (1 + q^2 + (1 + tq^4)Kh')$  and  $Kh_{\mathbb{F}_2}(L) = q^{s-1} (1 + q^2) (1 + (1 + tq^2)Kh')$  for even  $s = s(L)$  and non-negative-coefficients laurent polynomial  $Kh' = Kh'(L)$ .

**Conjecture 2.** For alternating knots  $s$  is the signature and  $Kh'$  depends only on  $tq^2$ .

**References.** Khovanov's arXiv:math.QA/9908171 and arXiv:math.QA/0103190 and DBN's

<http://www.ma.huji.ac.il/~drorbn/papers/Categorification/>.

More at <http://www.math.toronto.edu/~drorbn/Talks/UW0-040213/>