

Dror's Dream Map of Quantum Groups $\subset$ Knot Theory

Drinfel'd, Etingof-Kazhdan

Prehistoric

Hallucinations

about
Khovanov

C A T E G O R I F I C A T I O N



Some harsh reality.

EK mix tangles and braids
and algebras and Verma modules

Night Time Dreams

about
Etingof - Kazhdan $\mathcal{K} / \text{translucent vertices}$  $\hat{\mathcal{K}} / tv$  $gr \hat{\mathcal{K}} / tv$ Z / tv $\mathcal{A} / tv$ $\mathcal{K}_V = K(T)VG$ Knotted (trivalent?)
virtual knots $\hat{\mathcal{K}}_V$  $gr \hat{\mathcal{K}}_V$ $\mathcal{A}$ W specific
quantum
groups

Day Time Dreams

about Reshetikhin
- Turaev & quasi
triangular quasi
Hopf algebrasGens and rels? $\Rightarrow$  $\mathcal{K} = KTG$ Knotted Trivalent Graphs
with operations:

delete

unzip

#

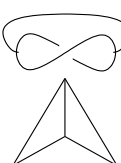
The unipotent completion

This is the Drinfel'd
theory of Associators!

Reality

of associators, FT
invariants and
co-commutative
quasi-Hopf
algebras

Generators



Relations

A 5 term relation,
a 6 term relation,
and lesser ones.

Definable subsets:

Genus g knots, ribbon knots, boundary links and more...

Why should we care?

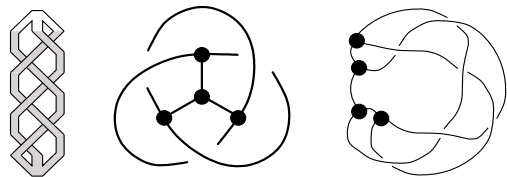
1. Usefulness!
2. Beauty!
3. Guidance!
4. Confidence!
5. Grothendieck-Teichmüller!



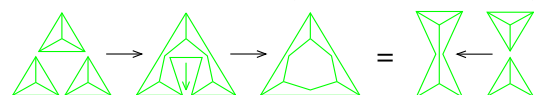
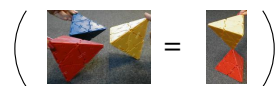
Fushimi-Inari

A La Carte Drawings

Knotted Trivalent Graphs (KTG's):

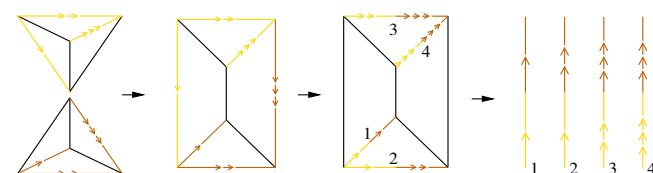
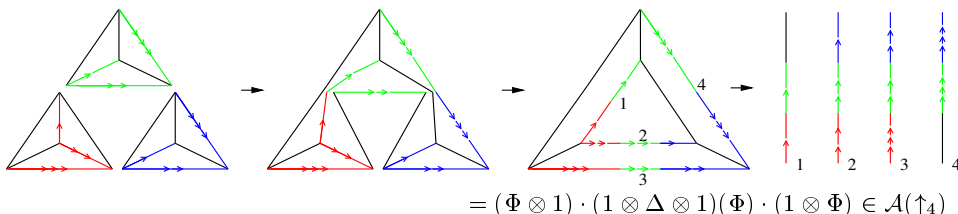


The 5-term relation:



Claim. With $\Phi := Z(\Delta)$, the above relation is equivalent to the Drinfel'd's pentagon equation.

Proof.



$$= (\Delta \otimes 1 \otimes 1)(\Phi) \cdot (1 \otimes 1 \otimes \Delta)(\Phi) \in \mathcal{A}(\uparrow_4)$$



Drinfel'd



Etingof



Kazhdan



J. Murakami



Ohtsuki



Kauffman



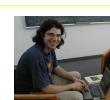
Goussarov



Polyak



Viro



D. Thurston



Haviv

More at <http://www.math.toronto.edu/~drorbn/Talks/Kyoto-0705/>