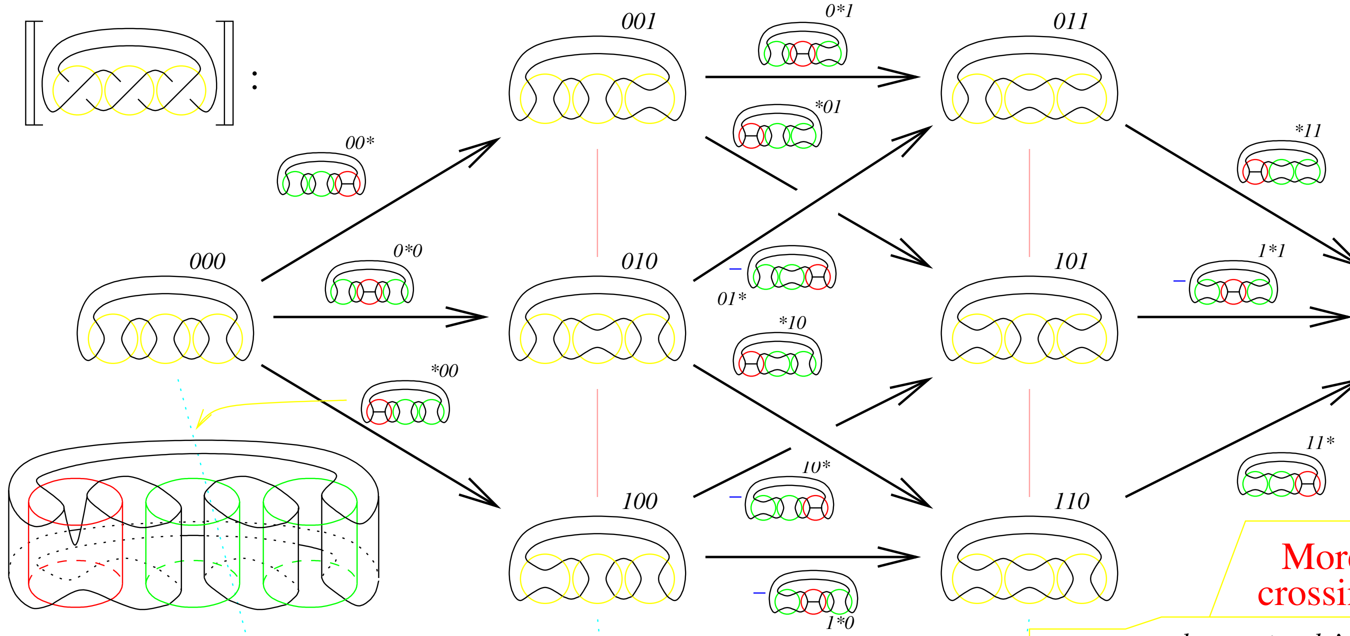
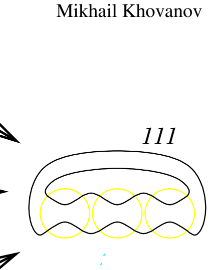
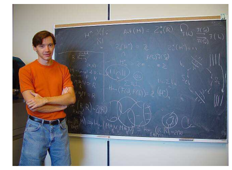
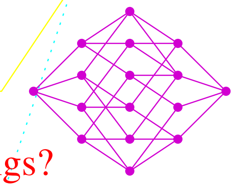
Khovanov Homology

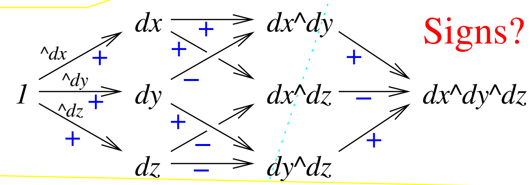


Mikhail Khovanov

More crossings?

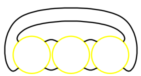
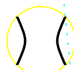
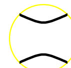


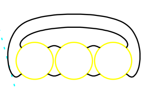
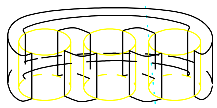
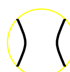
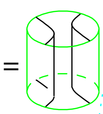
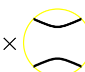
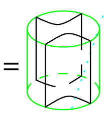
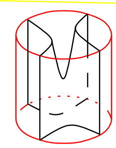
Signs?



What is it?

A cube for each knot/link projection;

Vertices: All fillings of  with  or with .

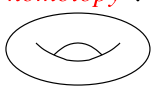
Edges: All fillings of $I\times$  =  with $I\times$  =  or with $I\times$  =  and precisely one .

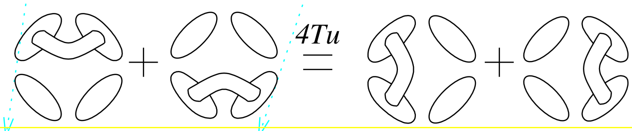
Where does it live?

In *Kom(Mat(<Cob> / {S, T, 4Tu})) / homotopy* :

Kom: Complexes *Cob: Cobordisms*

<...>: Formal lin. comb. *Mat: Matrices*

S:  *=0* *T:*  *=2*



Jones/Kauffman?

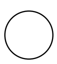
$$V^{\otimes 3}\longrightarrow (V^{\otimes 2}\oplus V^{\otimes 2}\oplus V^{\otimes 2})\{1\}\longrightarrow (V\oplus V\oplus V)\{2\}\longrightarrow V^{\otimes 2}\{3\}$$

$$(q+q^{-1})^3 \qquad 3q(q+q^{-1})^2 \qquad 3q^2(q+q^{-1}) \qquad q^3(q+q^{-1})^2$$

V. Jones

L. Kauffman

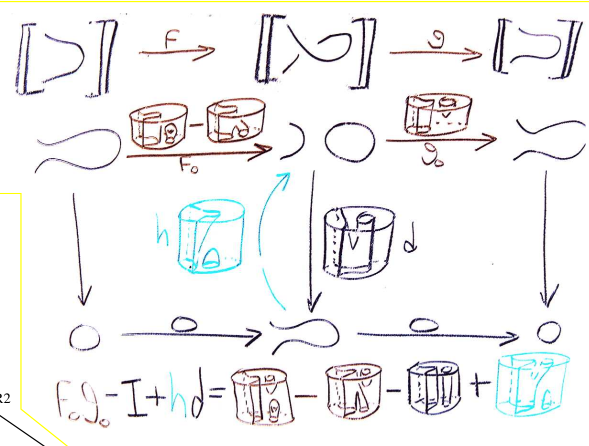
A TQFT takes it to a complex whose graded Euler characteristic is the Jones polynomial.

The key point:  → $V=\langle v_+,v_-\rangle,\quad \deg v_\pm=\pm 1$

$q\text{-}\dim V = q+q^{-1}$

But is it invariant?

(With similar proofs for R–II and R–III)

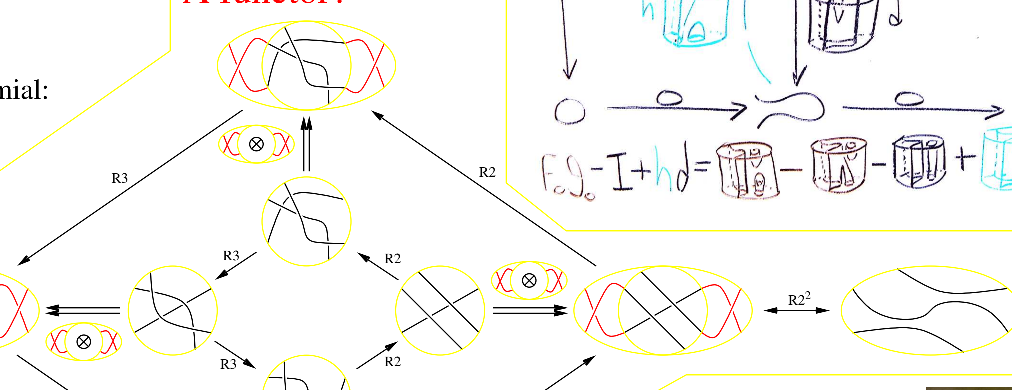
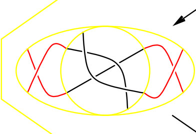
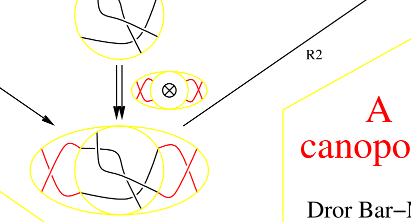


Why is it interesting?

1. It is stronger than the Jones polynomial.
2. It is less understood than the Jones polynomial:
   a. Does it have a topological interpretation?
   b. Does it have a "physical" interpretation?
   c. Does it also work for other quantum invariants?
   d. Does it work for manifolds and for knots in manifolds?
   e. Is there a relation with finite–type invariants?
   f. Does it work for "virtual knots"?
3. Jacobsson, Khovanov: It is a functor!!! (from knots and cobordisms to complexes and morphisms)

M. Jacobsson

A functor?

See http://www.math.toronto.edu/~drorbn/papers/Cobordism

A canopoly?

Dror Bar–Natan, Warszawa, July 2003.