# LINEAR ALGEBRAIC GROUPS (MAT 1110, WINTER 2017) HOMEWORK 2, DUE MARCH 1, 2017

FLORIAN HERZIG

**Problem 1.** Recall from class that a sequence of algebraic groups

$$1 \to K \xrightarrow{\phi} G \xrightarrow{\psi} H \to 1$$

is **exact** if

(a) it is exact as sequence of abstract groups and

(b) $0 \to \operatorname{Lie} K \xrightarrow{d\phi} \operatorname{Lie} G \xrightarrow{d\psi} \operatorname{Lie} H \to 0$ is an exact sequence of Lie algebras (i.e., of vector spaces).

(i) Show that $\phi$ is a closed immersion iff $\phi$ is injective and $d\phi$ is injective.

(ii) Suppose that $G$ is connected. Show that $\psi$ is separable iff $\psi$ is surjective and $d\psi$ is surjective.

(iii) Suppose that $G$ is connected. Deduce that $1 \to K \xrightarrow{\phi} G \xrightarrow{\psi} H \to 1$ is exact iff (a) and (b′) hold, where
(b′) $\phi$ is a closed immersion and $\psi$ is separable.

(iv) If the characteristic of $k$ is 0, show that (a) implies (b). (Hint: reduce to the case when $G$ is connected.)

**Problem 2.** Solve the following exercises from Springer's book: 3.2.10(2,3,4), 3.4.10(2), 4.4.11(3), 4.4.15(6)

**Update:** Please note the following in Springer, 3.2.10(2): for showing that $\phi$ injective implies $\phi^*$ surjective, please assume that the characteristic of $k$ is 0. (If it characteristic of $k$ is $p > 0$, consider the $p$-th power map $\mathbb{G}_m \to \mathbb{G}_m$.)

**Update:** Please note the following typo in Springer, 3.4.10(2): on the left-hand side it should say $(x, y)\cdot(x', y')$! (This construction comes from the ring of Witt vectors, there's even a way to extend it to an "algebraic ring"...)