MAT327 - Introduction to Topology

Summer 2019

Information at a Glance

Instructor	Ivan Khatchatourian	**Lecture times**	W1-3 and F12
Email	ivan@math.toronto.edu	**Lecture location**	MP137
Office	HU1027	**Office hours**	W4 and R1-3 in HU1018
TAs	Jinhui Li Armanpreet Pannu	**Tutorials**	T0101: F1 in BA2195 T0201: F2 in BA2195

Course website	http://uoft.me/mat327
Piazza	Signup: http://piazza.com/utoronto.ca/summer2019/mat327 Regular: http://piazza.com/utoronto.ca/summer2019/mat327/home

Contents

1 Goals and expectations

The first goal of this course is to provide a through and engaging introduction to point-set topology; its core concepts, its major theorems, and some of its other more interesting results. This subject is often maligned or thought of as boring, but I find fascinating and elegant. As a set theorist "by trade", I also intend to sprinkle some set theoretic flavouring over the proceedings.

The second, broader goal of this course is to convey a sense of how mathematicians go about creating an abstract theory like the theory of point-set topology; how we decide on definitions, how we formulate and prove theorems, and how we look past the technicalities of the definitions and understand the intuition behind them. Point-set topology is uniquely well-suited to this sort of exploration.

This course will necessarily delve into some deep, elaborate, and tricky mathematics, and you should expect the course to ramp up in difficulty throughout the summer. I am very excited to present this material to the students, and share my enthusiasm with them. My expectations for students are reasonable though. I expect students to engage with the course material thoughtfully, to talk to me and the TAs regularly, to develop a working understanding of the relevant definitions, statements major theorems, and their applications within the subject. In particular, I am not expecting to only pass research-ready, elite mathematicians. My goal is that students who engage actively with the material should do well.

The most important thing I expect of students entering this course is some degree of *mathematical maturity*. This is a tricky concept to define, but for example I expect students to be able to understand and manipulate mathematical definitions, to be comfortable manipulating sets, functions and other familiar objects, and to be able to write clear, correct proofs of simple things. Of course, we will exercise and hopefully substantially strengthen these skills as we go.

2 Topics

What follows is a list of planned topics for this course, in approximately chronological order. Covering all of these topics is an ambitious goal, and the list is subject to modification due to time constraints. Lecture notes will be grouped roughly in the same way.

- Topological spaces
- Bases of topologies
- Closed sets, closures, and density
- A crash course in countability in general, and second countability
- Sequence convergence, the Hausdorff property, and first countability
- Continuous functions, homeomorphisms, and topological invariants
- Subspaces
- Finite products of topological spaces
- The separation axioms (in particular, regularity and normality)
- Orders, order topologies, and ω_1
- The Axiom of Choice, and Zorn's Lemma
- Metric spaces, metrizability and Urysohn's Lemma
- Arbitrary products of topological spaces, and Urysohn's Metrization Theorem
- Compactness and the Heine-Borel Theorem
- Filters, ultrafilters, and Tychonoff's Theorem
- Connectedness
- Compactifications (one-point and Stone-Čech)

3 Textbooks

There is **no required textbook** for this class. I have created my own lecture notes and will be posting them publicly on the course website. With that said, please be aware of the following two books:

- *Topology (Second Edition)*, by James Munkres.

  The standard textbook for this course. It is easy to read and an excellent text for self-study. Note however that it orders topics and does some proofs differently than I intend to. I will not be following this book, but it will undoubedly be a useful resource. The other downside of this book: it costs about $125 (new from the bookstore).

- *Counterexamples in Topology*, by Steen and Seebach.

  A sort of "dictionary" of topological spaces that contains almost every definition from this course and much more. This is an invaluable reference that I still use frequently. A colleague and I used to joke that one should have this book on them at all times, in case a topological trivia question needs answering. That said, it is absolutely *not* suited for learning this material from scratch, and contains almost no proofs. The best part: it costs under $20.

4 Office hours

I will be holding three office hours per week throughout the course, **starting in the second week**. The tentative times for these are listed at the beginning of this document, and are subject to change pending discussion with the class about what times are convenient and pending room availability.

While you are never required to attend office hours, I consider them one of the most important aspects of the course. The students who do best tend to be the ones who come to office hours the most. You should attend whenever you can.

If you are unable to attend scheduled office hours you are welcome to email me and set up a special appointment.

5 The Big List of Problems

Point-set topology is a subject *rife* with "standard problems". Most of the problems and proofs you *must* complete in order to understand this material are very easily Googled, and solutions are very easily found (for example, one can easily obtain the solution to most any problem in Munkres' *Topology* in minutes). As a result it is extremely difficult to curtail cheating on homework. For this and a few other reasons, there will be **no graded homework assignments** in this course.

Instead, I will be providing a **Big List of Problems** on the course website, which will grow over time, sorted by topic and approximate difficulty. These are problems I have gathered from many sources, and will include most (but not all) exercises from the lecture notes. They will range in difficulty from easy to very hard. They will serve as topics of discussion in tutorials, and **will constitute a significant percentage of the problems appearing on quizzes, the midterm, and the final exam.**

I will not post solutions to all of these problems, because doing so would defeat the purpose of assigning them. I expect that students enrolling in this course will have understood by this point in their undergraduate mathematical careers that the valuable part of doing problems is sitting and thinking about them, not in reading their solutions. You will see solutions to some of them during tutorials and in quiz solutions, however. You are welcome (and encouraged) to work on these problems collaboratively, and to talk to me about them. I ask you not to post full solutions to problems on Piazza, as doing so would ruin the problems for people who have not thought about them yet.

6 Tutorials

Tutorials will constitute an essential part of this course, and should be thought of as no less important than the lectures. They begin **during in the second week** of the course. In particular, quizzes will be written during tutorials.

There are two tutorials, both led by **TBA**, at the following times and locations:

Section	Time	Location
T0101	F1	BA2195
T0201	F2	BA2195

7 Quizzes

Along with a midterm test and a final examination, students will be evaluated via **four, 50-minute quizzes, written in tutorials.** Students must write quizzes in the tutorials in which they are enrolled.

The currently planned dates for the quizzes are as follows.

Quiz	Date
Quiz 1	24 May
Quiz 2	7 June
Quiz 3	12 July
Quiz 4	2 August

Quizzes that are missed for legitimate reasons with valid documentation will have their weight transferred to the midterm or final exam.

8 Midterm

We will have one midterm test, 110 minutes in length, written in **BA1180**. The tentative date and time for the midterm is **Monday, 17 June, 6:10–8pm**.

This date and time are subject to change, pending room booking availability. Rest assured you will know the confirmed date, time, and location well in advance of the test.

A student who misses the midterm test due to illness must contact the instructor by **TBA**, and provide valid documentation to the instructor as soon as possible afterwards and no later than **TBA**. If such documentation is provided, the weight of the midterm will be shifted to the final exam or a makeup test will be administered, *at the instuctor's discretion.* If such documentation is not provided and/or the student has not made contact with the instructor by the above deadline, the student will receive a mark of 0 on the term test. **This is non-negotiable.**

9 Grading Scheme

The marking scheme for the course is as follows:

- 12.5% - highest quiz mark
- 7.5% - lowest quiz mark
- 10% each - remaining two quiz marks
- 20% - midterm test
- 40% - final examination, three hours in length

10 What to do if you miss a quiz or test

If you have an academic conflict with the midterm test (for example, a tutorial or a lab for a different course) and you provide documentation of this to the instructor at least one week before the test, a make-up test will be administered at an appropriate time. You should not have an academic conflict with any of the quizzes, since they are written in the tutorial in which you enrolled.

If you are unable to write one of the quizzes or the midterm test for legitimate reasons (such as severe illness), you will be accommodated provided that you (a) notify the instructor as soon as possible (no more than three days after the test); and (b) provide the appropriate documentation *in person* within a week of the date of the test. If you miss a quiz or the midterm due to severe illness or injury, you *must* use the University of Toronto Verification of Illness or Injury Form, found here:

http://www.illnessverification.utoronto.ca/

11 Accessibility

Accessibility Services collaborates with students, instructors, volunteers and staff to provide accommodations to students with documented disabilities in order that they may participate as equals in all academic matters. If you have any accessibility-related questions or concerns, you are encouraged to contact and/or register with Accessibility Services. For more information, please visit:

http://www.accessibility.utoronto.ca

12 Academic integrity

All students at the University of Toronto are expected and required to be familiar with this institution's policies on academic integrity. In this course in particular, any student found guilty of academic misconduct during a quiz will be penalized with a grade of 0% on the quiz in question, and that quiz will count for 10% of his or her final course mark (irrespective of the grades on the other quizzes).

Students are encouraged to refresh themselves on the details of what constitutes academic misconduct at the following two links:

http://www.artsci.utoronto.ca/newstudents/transition/academic/plagiarism
http://www.artsci.utoronto.ca/osai/The-rules/what-is-academic-misconduct

13 Important dates

Here are some important dates for this course. A few of these dates are subject to change, as discussed in preceding sections of this document.

Note that this does list does not mention holidays or any other events that have no bearing on this course.

Date	**Significance**
Mon, 6 May	Summer term classes begin
Wed, 8 May	Our first lecture
Sun, 12 May	Last day to enrol in this course on ACORN
Fri, 17 May	First meeting of both tutorial sections
Fri, 24 May	Quiz 1 written in tutorials
Fri, 7 June	Quiz 2 written in tutorials
Fri, 14 June	Our last lecture of the first term
TENTATIVE! Tue, 18 June	Midterm test
Wed, 19 June - Fri, 28 June	Summer reading break (**no classes**)
Wed, 3 July	Our first lecture of the second term
Fri, 12 July	Quiz 3 written in tutorials
Mon, 15 July	Last day to drop the course without academic penalty
TBA	August exam schedule is posted
Fri, 2 August	Quiz 4 written in tutorials
Fri, 9 August	Our last lecture (and last tutorial)
Mon, 12 August	Last day of summer term Last day to request late withdrawal
Thu, 15 August - Thu, 22 August	Summer exam exam period