HOMEWORK SET #5: DUE NOVEMBER 26

(1) Alice and Bob play the following game. There are 2 piles, consisting of 100 and 200 matchsticks respectively. With Alice going first, they alternate removing 1 or 2 matchsticks from one of the piles. The winner is the person who takes the last matchstick. Who has a winning strategy?

(2) Alice and Bob play the following games. There are n stones in a pile, with the numbers $1, 2, 3, \dots, n$ written on them, one number per stone. With Alice going first, they take turns removing stones from the pile. However, a player may not remove a stone if it leads to two consecutive numbers being removed from the pile. I.e. if Bob takes the stone with the number 5 on it, then from then on noone may take the stones with the numbers 4 or 6 on them.
   (a) Prove that for $n = 4$, Bob has a winning strategy.
   (b) Prove that for $n = 101$, Alice has a winning strategy.

(3) Alice and Bob play a game on an 4×4 chessboard in which the first player places a knight on a chessboard, and then they alternate turns moving the knight to a square that it has not previously occupied. The player that cannot move loses. Who has a winning strategy?

(4) Alice and Bob play a game on a 3×3 board. Beginning with Alice, they take turns placing either a white or black checker on an unoccupied square of the board. The game ends after 9 turns. At the end of the game, Alice obtains 1 point for ever row, column, or diagonal containing an odd number of black checkers, and Bob gets a point for every row, column, or diagonal with an even number of black checkers. The winner is the person with more points, and they tie if they each get 4 points. Prove that Alice has a winning strategy.