MAT 495 HOMEWORK 3: GAME THEORY

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Write up and submit any 4 problems *with proofs* by Friday, October 23rd. Homework should be submitted to Asif Zaman's Mailbox, in the Bahen center, Math office, or in class. Points will be deducted for missing cases, gaps in the proof, algebra mistakes, and errors in reasoning, so please write very carefully!

If you can't solve 4 problems, you can submit partial work, but please do not submit work on more than 4 problems.

1. Problems

(1) Alice and Bob are playing a game on a 4×4 chessboard, with Alice going first. On a players turn, they paint one of the squares(which has not yet been painted). The first person who finishes painting a 2×2 sub-board loses (there are 9 such sub-boards). Who has a winning strategy?

(2) A knight is place on an 8×8 chessboard by Alice. Then Bob moves the knight to a different square, using a legal knight's move. The players then alternate moving the knight using legal knights move, with the restriction that a knight cannot be moved to a square it has previously visited. The player who cannot make a move loses. Determine who has a forced win.

(3) On the lower left corner of a chessboard, there is a king. Alice and Bob take turns moving the king either one space to the right, one space upward, or one space upward and to the right. The player who reaches the upper right corner is the winner, and Alice goes first. Determine who has a winning strategy. (Try small cases!)

(4) You and I play a game using an ordinary deck of 52 cards(there are 4 suits and 13 cards in each suit). We begin with me picking any 5 cards from the deck (which is turned face up). The you pick any 5 cards from the 47 cards remaining. Then I may throw away 0,1,2,3,4, or 5 cards from my hand and replace them with new cards from the deck. Then you may throw away 0,1,2,3,4, or 5 cards from your hands and replacing them with new cards from the deck (you may not take the cards I threw away!). If my hand is stronger than yours, I win the game. Otherwise, you win (even if our hands are of equal strength). Who has a winning strategy?

(5) Alice and Bob play a game with two piles of chips, one with 26 chips and one with 25 chips. Alice and Bob take turns (Alice goes first)

doing one of the following 3 moves: remove a chip from one of the bags, remove a chip from both of the bags, or move a chip from one of the bags to the other bag. The player whose move leaves both bags empty wins.

Note that the game COULD go on forever! Prove that Alice has a winning strategy (that is, she can play in such a way that the game will end in a finite number of moves and she will be the winner)."

(6) (The WOW game) Alice and Bob play a game on a row of 7 squares, initially all of them empty. With Alice going first, the players take turn writing "W" or "O" in one of the squares, never writing twice in the same square. The player who first succeeds in having 3 consecutive squares spell "WOW" wins. If the whole board gets filled up without an "WOW" appearing, the game is a draw. Determine who has a winning strategy.
(7) (Hard!) Who wins the WOW game if instead of playing on a row of 7 squares we play on a row of 2014 squares?
(8) (Hard!)There are 500 frogs in a pond. Certain frogs are friends. It is known that no matter how you split the frogs up in 250 pairs of frogs, at least one of those pairs will consist of a pair of frogs that are not friends. You and I play a game alternatively picking frogs up out of the point and placing them in our pool. I start by picking any frog, but afterwards a a frog can only be picked up if he is friends with the frog last that was picked up, and no frog can be picked up twice. The person who cannot pick up a frog loses. Prove that I have a winning strategy (for example, if NONE of the frogs are friends, I can pick up any frog and then you cannot make a move, so you lose).