MAT137

- This week, we have classes on Monday, Wednesday, and Thursday.

Asymptotes

Definition

We say that $x = a$ is a *vertical asymptote* for a function f if at least one of the following is true

$$\lim_{x \to a^+} f(x) = \infty, \ , \lim_{x \to a^-} f(x) = \infty, \ \lim_{x \to a^+} f(x) = -\infty, \ \lim_{x \to a^-} f(x) = -\infty$$

Definition

We say that $y = c$ is a *horizontal asymptote* for a function f if at least one of the following is true

$$\lim_{x \to \infty} f(x) = c, \ , \lim_{x \to -\infty} f(x) = c$$

True or False

1. If $y = c$ is a horizontal asymptote for f, then the graph of f never crosses the line $y = c$.
2. If $x = a$ is a vertical asymptote for f and f is continuous on its domain, then the graph of f never crosses the line $x = a$.
3. If $f(x) = g(x)/h(x)$ and $h(a) = 0$, then $x = a$ is a vertical asymptote for f.
4. If $f(x) = g(x)/h(x)$ where both g and h are polynomials of the same degree, then f has a horizontal asymptote.
5. The function $f(x) = \frac{\sin x}{x}$ has a horizontal asymptote.
6. Every function has only finitely many vertical asymptotes.
7. If $f(x) = g(x)/h(x)$ where g and h are polynomials and the degree of h is bigger than the degree of g, then f has a horizontal asymptote.
8. If $f(x) = g(x)/h(x)$ where g and h are polynomials and the degree of h is bigger than the degree of g, then f has a vertical asymptote.

Vertical tangents and cusps

Definition

Let f be a continuous function which is differentiable everywhere except at a. We say that the graph f has a *vertical tangent* at $(a, f(a))$ if

$$\lim_{x \to a} f(x) = \infty \quad \text{or} \quad \lim_{x \to a} f(x) = -\infty$$

Definition

Let f be a continuous function which is differentiable everywhere except at a. We say that the graph f has a *vertical cusp* at $(a, f(a))$ if

$$\lim_{x \to a^+} f(x) = \infty \quad \text{and} \quad \lim_{x \to a^-} f(x) = -\infty$$

or vice versa.

What is an example of a function with a vertical tangent? What is an example of a function with a vertical cusp?